

Loudspeaker Rocking Modes (Part 1: Modeling)

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The rocking of the loudspeaker diaphragm is a severe problem in headphones, micro-speakers and other kinds of loudspeakers causing voice coil rubbing which limits the maximum acoustical output at low frequencies. The root causes of this problem are small irregularities in the circumferential distribution of the stiffness, mass and magnetic field in the gap. A dynamic model describing the mechanism governing rocking modes is presented and a suitable structure for the separation and quantification of the three root causes exciting the rocking modes is developed. The model is validated experimentally for the three root causes and the responses are discussed conforming a basic diagnostics analysis.

1. INTRODUCTION

Rocking modes are natural vibration patterns of the loudspeaker, producing undesired rotational vibrations. Loudspeakers drivers with substantial irregularities produced in the manufacturing/assembly process and drivers subjected to asymmetric acoustics loads exhibit this phenomena. This rocking behaviour becomes more critical in small drivers such as headphones or micro-speakers, where small irregularities in the stiffness, mass and magnetic field distributions, can affect dramatically the dynamic behaviour of these tiny structures. Even when high precision in the production process and low tolerances in the assembly are reached, some of these drivers present substantial excitation of rocking modes. One of the most important problems associated is the rub&buzz effect which limits the acoustic output before the normal mechanical or thermal mechanisms do. Since some of the causes of the rocking modes are nonlinear like: $B_l(x)$ and $K_{ms}(x)$, this problem can appear only at high amplitudes, for this reason, drivers that are very stable at low levels can become unstable at high operation levels.

The detection and identification of this rocking behaviour is crucial in the loudspeaker development process, since it will reveal the root cause of these particular symptoms allowing the engineer to take the necessary actions to eliminate or mitigate instability problems.

The main idea is to develop an optimal model that can explain the generation of rocking modes and that allows to quantify the magnitude and the direction of the mentioned problems. This information needs to be easy to interpret and should be close to the physical phenomena experienced by engineers and designers working daily on loudspeaker development and manufacturing. Since the natural modes are the simplest

way to express how a structure vibrates, expressing the amount of excitation of those modes is a very intuitive approach that provides valuable information for understanding problems.

This paper is distributed as follows: First the physical mechanisms and the consequences of each root cause are described. After that, the system of equations are presented, the separation of the different causes as excitation mechanisms are developed and the simplified model is derived. In chapter four the modal analysis and the projection of the root-causes moments is described. Finally a set of validation experiments is discussed.

2. PHYSICAL MODEL

2.1. Rigid body motion assumption

The rocking mode behavior of a loudspeaker can be considered as a low frequency mechanism totally described by the equations of motion derived for the lumped parameter model [1] coupled to another set of equations describing the tilting angle around a determined rotation axis. In this context, only the suspension parts are deformed under the effect of applied forces and moments. This assumption is fulfilled by all kind of electro-dynamical transducers composed by a rigid diaphragm, voice coil and former attached to slightly softer suspension parts. In the case of micro-speakers and headphones, the diaphragm and the surround are made of the same material and the diaphragm needs to be deformed to allow the rotational degree of freedom. In this special cases the diaphragm will be taken as the larger surface vibrating with some negligible deformation. The frequency range for the rocking mode analysis is valid only below the first breakup mode of the diaphragm.

2.1.1. System of coordinates

The selection of the system of coordinates to be used in the derivation of the equations of motion will determine the usefulness of the model for the separation of the root causes of rocking modes. In the model presented in [2], the system of equations is derived assuming that the pivot point of the rotations along x and y coincides exactly with the center of gravity of the driver. Based on this coordinate system, all the moments associated to a shifting of the center of gravity of the diaphragm around the pivot point are zero, in other words the equations are dynamically decoupled [3], as a consequence the causes due to mass problems cannot be independently extracted. Since the equations will describe exactly the same behaviour independent of the system of coordinates or variables used, by using the mentioned coordinates, all the physical causes will be attributed to a static coupling terms produced in the suspension parts.

For performing accurate root-cause analysis of rocking modes including the quantification of problems associated to Mass, Stiffness and B-field distributions, the model presented in this paper is extended to include the dynamic and static couplings as well as the moments terms associated to the excitation force.

Starting from the contributions made by MacLachlan in [4] and later by Bright in [2] and [5] the rocking mode model can be developed by using the equations of rigid body mechanics.

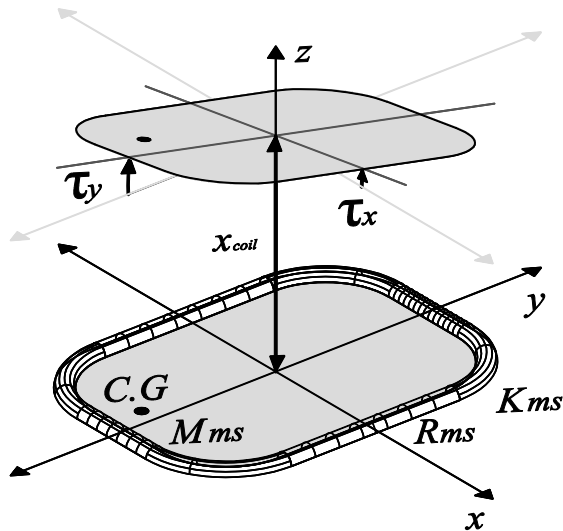


Figure 1 State variables describing rocking mode of a loudspeaker.

The complete behavior of a loudspeaker exhibiting rocking modes can be described by three states; one

translational x_{coil} describing the position of the center of the coil along the z axis and two rotational τ_x and τ_y describing the tilting angles around the y and x axis respectively, see Figure 1.

The mechanical mass M_{ms} is assumed to be lumped at the center of gravity of the diaphragm $C.G$ and the total stiffness K_{ms} including viscoelastic effects and R_{ms} mechanical resistance are associated to the suspension parts.

2.1.2. Total equivalent moments

The imperfections produced in the production process, such as inhomogeneous distribution of the material or thickness over the diaphragm, geometrical or material changes along the suspension parts and differences of the magnetic field in the gap usually are located in different angles and at different distances from the center of the driver. As they can be seen as a moments acting around an arbitrary rotation angle (coincident with the direction of the rocking modes), all its contributions can be collapsed to a one equivalent moment producing the same effect. The tilting angle produced by the actions of the moments around the rotation axis of the rocking mode is called τ_a . The graphical representation of this concept is depicted in Figure 2.

The force f_{sym} corresponds to a part of the total applied mechanical force coming from the motor that produces only displacement of the diaphragm in the z -direction (pure excitation of fundamental mode), is called symmetric force because it does not produces any tilting of the moving mass. The other moments are results of some asymmetrical forces acting on the system.

The total moment produced by problems in the magnetic field μ_{BTa} is located at some arbitrary direction at the distance where the voice coil former and the diaphragm are in contact. In the other hand, the total moment μ_{kTa} generated due to irregular stiffness distribution will appear at the contact points between the suspension parts and the moving mass.

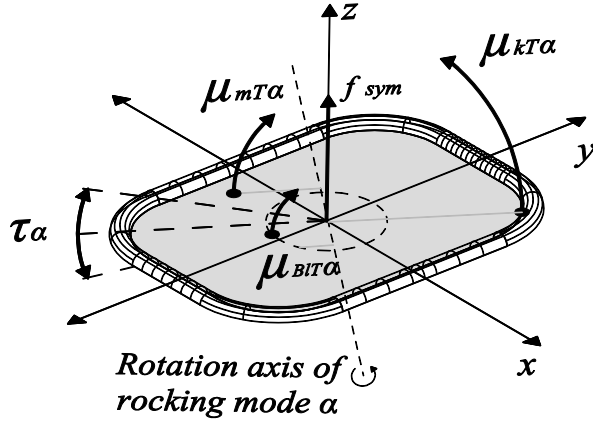


Figure 2 Separated total moments exciting a rocking mode

The total moment contributing to the rocking mode due to a mass problem $\mu_m T \alpha$ can be seen as the effect of a shifting of the center of gravity $C.G$ of the moving mass with respect to the geometrical center. This moment can be generated as well by some excess of glue in the junctions with the suspension parts and is taken into account only if the origin of the system of coordinates is located at the geometrical center of the driver.

2.2. Root causes and excitation terms

In the previous sections, it has been stated that the rocking modes in loudspeakers will be activated only if some moments around the axis crossing the center of the driver exists. In other words, any kind of nonsymmetrical force acting on the diaphragm will cause a tilting vibration known as rocking mode. They can be generated by irregular mass and stiffness distribution on the moving parts (passive excitation) or from the imperfections in the Bl distribution in the gap (active excitation).

2.2.1. Mass Unbalance

This problem is related with the shifting of the center of gravity of the moving parts out of the geometrical center. It can happen due to non-uniform material distributions on the driver components, thickness variations, excess of gluing in a certain direction, etc. The mass difference Δ_m between two opposite regions of the loudspeaker, will produce a shifting of the center of gravity $C.G$ which consequently generates a moment μ_m at a distance ε around the axis crossing the geometrical center which will produce a tilting angle τ_m as shown in Figure 3.

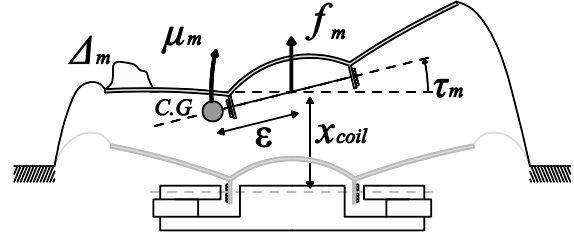


Figure 3 Tilting generation due to a mass unbalance

The equilibrium of forces and moments acting on a infinitesimal part of the diaphragm mass dm located at $C.G$ are:

$$f_m = \int (\ddot{x}_{coil} + \varepsilon \ddot{\tau}_m) dm \quad (1)$$

$$= M_{ms} \ddot{x}_{coil} + \Delta_m \ddot{\tau}_m$$

$$\mu_m = \int (\ddot{x}_{coil} + \varepsilon \ddot{\tau}_m) \varepsilon dm \quad (2)$$

$$= \Delta_m \ddot{x}_{coil} + I_c \ddot{\tau}_m$$

Assuming that the distributed mass of the loudspeaker $M_{ms} = \int dm$ is lumped at center of gravity $C.G$ the parameter $\Delta_m = \varepsilon M_{ms}$ quantifies the difference of mass between two extremes of the loudspeaker in the direction of the distance ε from the geometrical center of the moving assembly. The moment of inertia is defined as $I_c = \varepsilon^2 M_{ms}$ [3]. Two effects are produced here; a force f_m accelerating the diaphragm up and down $d^2 x_{coil} / dt^2$ and a moment μ_m acting around the center of the driver producing a tilting angle τ_m . Note that the two coordinates x_{coil} and τ_m are coupled through the parameter Δ_m . If there is no shifting of the center of gravity $\varepsilon = 0$, the moment exciting the tilting angle is zero and the only remaining force will produce a perfect transversal motion.

The amplitude of this tilting angle will depend on the amount of shifting of the center of gravity produced by the irregular mass distribution. The mass problem is a dynamic process that becomes dominant above the fundamental resonance frequency of the loudspeaker where the voice coil acceleration is high.

2.2.2. Stiffness asymmetry

Any kind of defect or irregularity which modifies locally the restoring force of the suspension along the peripheral lines where the diaphragm is attached, can be seen as a difference of the stiffness Δ_k between two opposite contact points. Note that the total stiffness distribution acting on x_{coil} should be equivalent to K_{ms} .

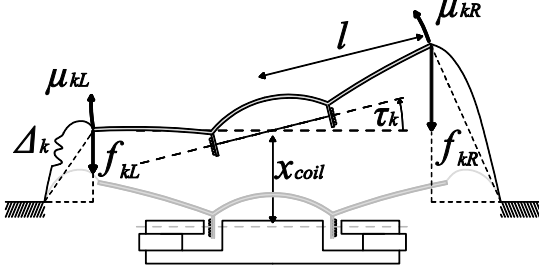


Figure 4 Tilting generation due to stiffness asymmetry

Computing the forces and moments for the simplified 2D case shown in Figure 4:

$$f_k = f_{kL} + f_{kR} = K_{ms} x_{coil} + K_0 \tau_k \quad (3)$$

$$\mu_k = K_0 x_{coil} + K_\tau \tau_k \quad (4)$$

The forces f_{kL} and f_{kR} represent the distributed vertical component of the restoring force provided by the suspension parts. μ_{kL} and μ_{kR} the moments around the center of the driver produced by the stiffness. The parameter $K_\tau = K_{ms} l^2$ is called the rotational stiffness around the rotational axis and $K_0 = K_{ms} l \Delta_k$, l is the distance from the center to the contact point with the surround. This irregularity will generate a total moment μ_k around the axis crossing the center of the driver producing a tilting τ_k .

Note that if the suspension parts exhibit some kind of problem, its effect will be evidenced even if there is no movement of the driver. For this reason this kind of tilting generation can be called static and is mainly active below the fundamental resonance frequency

2.2.3. B-field inhomogeneity

The force factor Bl determines the amount of mechanical force that a loudspeaker motor can produce from an applied current. This parameter depends locally on the amount of magnetic field B crossing the voice coil with length l . Several reasons can cause variations of the magnetic field along the gap, such as non-homogenous

magnetization, non-constant gap width due to shifted positions of the iron parts, etc. The non-asymmetrical transduction factor Δ_{Bl} can produce substantial changes of the mechanical force generated at different angles on the voice coil producing a moment μ_{Bl} around the rotation axis at a distance d_{coil} , contact point of the coil with the moving parts. The action of this moment will produce a tilting angle τ_{Bl} of the voice coil as seen in Figure 5.

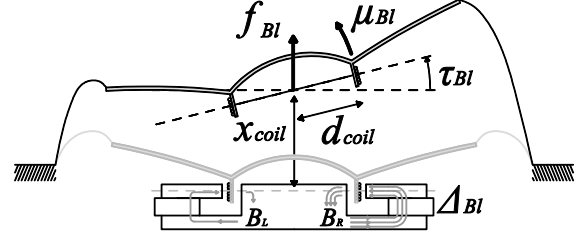


Figure 5 Tilting generation due to B-field inhomogeneity

The force and the moments produced by this problem are:

$$f_{Bl} = Bli \quad (5)$$

$$\mu_{Bl} = \Delta_{Bl} i d_{coil} \quad (6)$$

This root cause is related to the input current of the loudspeaker driver and it will excite the rocking resonators actively depending on its position over the voice coil circumference and the mode shape. This is active broadband but presents a dip at the resonance frequency due to the decreasing of current flowing through the voice coil.

3. ROCKING MODE MECHANISMS

3.1. Electromechanical system

By computing the equilibrium of forces acting in the z -direction and the moments around the x and y axis and coupling the electrical part of the driver through the current $i(t)$, the next set of motion equations can be derived:

$$\begin{bmatrix} M_{ms} & \Delta_{mx} & \Delta_{my} \\ \Delta_{mx} & I_{cx} & \Delta_{\tau}^m \\ \Delta_{my} & \Delta_{\tau}^m & I_{cy} \end{bmatrix} \begin{bmatrix} \dot{x}_{coil}(t) \\ \dot{\tau}_x(t) \\ \dot{\tau}_y(t) \end{bmatrix} + \begin{bmatrix} R_{ms} & 0 & 0 \\ 0 & R_{cx} & 0 \\ 0 & 0 & R_{cy} \end{bmatrix} \begin{bmatrix} x_{coil}(t) \\ \tau_x(t) \\ \tau_y(t) \end{bmatrix} + \begin{bmatrix} K_{ms} & \Delta_{kx} & \Delta_{ky} \\ \Delta_{kx} & K_{cx} & \Delta_{\tau}^k \\ \Delta_{ky} & \Delta_{\tau}^k & K_{cy} \end{bmatrix} \begin{bmatrix} x_{coil}(t) \\ \tau_x(t) \\ \tau_y(t) \end{bmatrix} = \begin{bmatrix} f_{sym} \\ \mu_{Blx} \\ \mu_{Bly} \end{bmatrix} = \begin{bmatrix} Bl \\ \Delta_{Blx} \\ \Delta_{Bly} \end{bmatrix} i(t) \quad (7)$$

where M_{ms} , R_{ms} and K_{ms} corresponds to the mechanical parameters of the driver, the product Bl is the force factor and they can be measured by using the conventional techniques [6] or [1].

The constant parameters Δ_{mx} and Δ_{my} are the terms coupling the translational displacement with the two rotational vibrations and Δ_{τ}^m represents the coupling

between the two rotational degree of freedom. These parameters are related to the position of center of gravity of the diaphragm and they are called *dynamic couplings* since they will be only active if there exists acceleration of the model states. Note that any mass problem present in the real driver will be coded by those parameters.

The parameters Δ_{kx} , Δ_{ky} and Δ_{τ}^k play the same role as in the mass matrix, but for the stiffness. Since they are directly multiplied by the states and not by some of its derivatives, they are called static couplings. Any kind of non-uniform distribution of the stiffness of the driver will be described by these terms.

The moments μ_{Blx} and μ_{Bly} exciting a tilting along the x and y axes are depending on the parameters Δ_{Blx} and Δ_{Bly} containing the information about the irregular B-field distribution in the gap.

The diagonal terms $I_{cx,y}$, $R_{rx,y}$ and $K_{rx,y}$ are the mass moment of inertia, the mechanical damping of the rotational vibration and the rotational stiffness of the diaphragm about the x and y axes.

Equation (7) can be transferred into Laplace domain as:

$$s^2\mathbf{M}\mathbf{x} + s\mathbf{C}\mathbf{x} + \mathbf{K}\mathbf{x} = \mathbf{B}I(s) \quad (8)$$

where the state vector $\mathbf{x} = [X_{coil}(s) T_x(s) T_y(s)]^T$ and the matrices \mathbf{M} , \mathbf{C} and \mathbf{K} represent the mass, mechanical resistance and stiffness, respectively. The vector \mathbf{B} represents the components of the electro-dynamical transduction factor.

For circular drivers, it has been shown in [2] that the stiffness terms can be expressed analytically as Fourier series of sinusoidal functions around the angle in the circular coordinate called *stiffness distribution functions*. In the case of the model presented here, this approach needs to be generalized since this model is intended to be applied to loudspeakers with arbitrary shapes.

The energy dissipation of the rotational (tilting) vibration is usually attributed to visco-thermal effects present in the gap and the mechanical resistance of the suspension. For simplicity, the damping matrix is chosen to be diagonal, neglecting coupling terms due to the velocity of the model states.

3.2. Direct excitation and feedback sources

The model described by Equation (7) is a fully coupled system of differential equations which is rather complex and difficult to interpret. A more understandable form can be obtained by separating the inertia and elasticity terms producing moments around the rotational axis of the diaphragm as feedback sources added to the direct excitation from the motor.

Separating the diagonal terms \mathbf{M}_D and \mathbf{K}_D of the mass and stiffness matrices from the non-diagonal (coupling) terms \mathbf{M}_Δ and \mathbf{K}_Δ gives

$$\left(s^2\mathbf{M}_D + s\mathbf{C} + \mathbf{K}_D\right)\mathbf{x} = \mathbf{B}I - s^2\mathbf{M}_\Delta\mathbf{x} - \mathbf{K}_\Delta\mathbf{x} \quad (9)$$

It can be clearly seen that the two new terms subtracted from the direct source and multiplied by the state vector can be considered as feedback sources of the system conformed by the diagonal terms on the left side of (9).

If the diagonal terms are grouped into a transfer function matrix \mathbf{H}_τ and the excitation terms associated to mass and stiffness are called $\mathbf{F}_{\Delta m}$ and $\mathbf{F}_{\Delta k}$ the full coupled differential equation takes the form:

$$\mathbf{H}_\tau\mathbf{x} = \mathbf{F}_{\Delta Bl} - \mathbf{F}_{\Delta m} - \mathbf{F}_{\Delta k} \quad (10)$$

Equation (10) describes three different sources corresponding to the mass, stiffness and Bl root-causes, activating a set of independent (decoupled) second order systems. A schematic representation of the derived Equation (10) is presented in Figure 6.

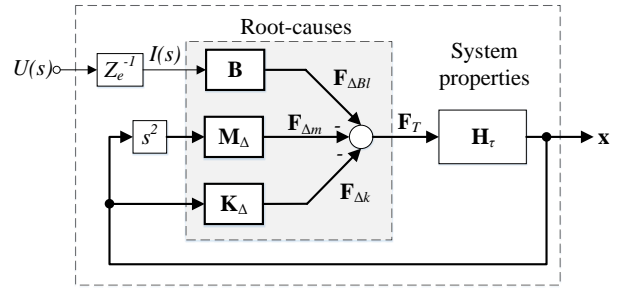


Figure 6 Mass and stiffness problems as feedback sources and magnetic excitation as a direct term.

The mechanisms governing rocking modes in loudspeaker diaphragms consist of a SIMO (single input multi output) system with an internal feedback structure (dominated by X_{coil}) through terms associated with mass and stiffness irregular distributions, activating a set of three second order resonators. The Z_e^{-1} is the electrical impedance converting the input voltage $U(s)$ into the current $I(s)$. The 2nd-order differentiator s^2 transforms the state vector into the acceleration. The terms associated with Bl inhomogeneities are the direct forces and moments depending on the input current $I(s)$.

From the physical perspective, the only way to excite the tilting angles of the diaphragm is by using a motor with some inhomogeneities in the magnetic field, these introduce energy directly on the resonators. In the case of the mass and stiffness sources, they do not contribute actively with the total energy applied to the driver, but they redistribute the energy, that is supposed to excite

only the piston mode, into the other two tilting components. For this reason they appears as a feedback sources in Figure 6.

Note that the total excitation vector \mathbf{F}_T is composed by the complex contributions of the translational and rotational states multiplied with the different root-causes terms. A detailed analysis is required to determine the relevant terms in practical diagnostics analysis.

3.3. Detailed moment generation and root-causes separation

The aim of this section is to describe the detailed derivation of a set of equations equivalent to equation (10) to provide a more intuitive idea of the generation

$$\begin{bmatrix} H_{xcoil}^{-1}(s) & 0 & 0 \\ 0 & H_{\alpha}^{-1}(s) & 0 \\ 0 & 0 & H_{\gamma}^{-1}(s) \end{bmatrix} \begin{bmatrix} X_{coil}(s) \\ T_x(s) \\ T_y(s) \end{bmatrix} = \begin{bmatrix} Bl \\ \Delta_{Blx} \\ \Delta_{Bly} \end{bmatrix} I(s) - s^2 \begin{bmatrix} 0 & \Delta_{mx} & \Delta_{my} \\ \Delta_{mx} & 0 & \Delta_{\tau}^m \\ \Delta_{my} & \Delta_{\tau}^m & 0 \end{bmatrix} \begin{bmatrix} X_{coil}(s) \\ T_x(s) \\ T_y(s) \end{bmatrix} - \begin{bmatrix} 0 & \Delta_{kx} & \Delta_{ky} \\ \Delta_{kx} & 0 & \Delta_{\tau}^k \\ \Delta_{ky} & \Delta_{\tau}^k & 0 \end{bmatrix} \begin{bmatrix} X_{coil}(s) \\ T_x(s) \\ T_y(s) \end{bmatrix} \quad (11)$$

The diagonal terms of the left side matrix are the inverse of the transfer functions between the output states of the model and the total input excitation of each one.

The transfer function:

$$H_{xcoil}(s) = \frac{1}{s^2 M_{ms} + s R_{ms} + K_{ms}} \quad (12)$$

is the ratio between the mechanical symmetrical force $f_{sym} = BlI(s)$ produced by the loudspeaker motor and the voice coil displacement $X_{coil}(s)$. The other two transfer functions:

$$H_{\alpha}(s) = \frac{1}{s^2 I_{ci} + s R_{\alpha} + K_{\alpha}} \quad (13)$$

where $i \in \{x, y\}$ are the ratios between the tilting components $T_i(s)$ and the total moments $\mu_i(s)$ activating them. This resonators are introduced here as auxiliary transfer functions which can describe the tilting oscillations of the diaphragm exactly coincident with the x - y directions of the coordinate system. They should not be confused with the rocking resonators associated to the dominant rocking mode and its orthogonal counterpart, which can be generated in any direction.

The coupling terms $s^2 \Delta_{mi}$ and Δ_{ki} determine the amount of interaction between the voice coil displacement and the tilting angles. These terms are dominant in practice.

In the other hand the coupling terms $s^2 \Delta_{\tau}^m$ and Δ_{τ}^k determine the interaction between the two rotational states. These terms are usually negligible in practice.

Solving equation (11) the transversal displacement $X_{coil}(s)$ and the tilting angles $T_x(s)$ and $T_y(s)$ can be described by second-order resonators and

mechanisms of the rocking modes in loudspeakers, making transparent the relation between the physical causes and the symptoms observed in real drivers. In the same analysis it is possible to determine the important terms and to neglect those which do not contribute significantly on the practical interpretation and diagnostics.

The separation of the diagonal and non-diagonal terms leads to the following equation:

$$X_{coil}(s) = H_{xcoil}(s) f_T(s) \quad (14)$$

$$T_x(s) = H_{\alpha}(s) \mu_{Tx}(s) \quad (15)$$

$$T_y(s) = H_{\alpha}(s) \mu_{Ty}(s) \quad (16)$$

the total transversal excitation force

$$\begin{aligned} f_T(s) &= f_{sym} - f_m - f_k \\ &= BlI(s) - s^2 \boldsymbol{\tau}^T \boldsymbol{\Delta}_m - \boldsymbol{\tau}^T \boldsymbol{\Delta}_k \end{aligned} \quad (17)$$

and the total excitation moments in x and y directions:

$$\begin{aligned} \mu_{Tx}(s) &= \mu_{Blx} - \mu_{mx} - \mu_{kx} \\ &= \Delta_{Blx} I(s) - s^2 \mathbf{x}_x^T \boldsymbol{\Delta}_{mx\tau} - \mathbf{x}_x^T \boldsymbol{\Delta}_{kx\tau} \end{aligned} \quad (18)$$

$$\begin{aligned} \mu_{Ty}(s) &= \mu_{Bly} - \mu_{my} - \mu_{ky} \\ &= \Delta_{Bly} I(s) - s^2 \mathbf{x}_y^T \boldsymbol{\Delta}_{my\tau} - \mathbf{x}_y^T \boldsymbol{\Delta}_{ky\tau} \end{aligned} \quad (19)$$

Each excitation force and moment comprise three components corresponding to root causes: mass, stiffness or Bl .

The vectors $\boldsymbol{\tau}$, \mathbf{x}_x and \mathbf{x}_y are called state sub-vectors and are defined as:

$$\boldsymbol{\tau} = \begin{Bmatrix} T_x \\ T_y \end{Bmatrix} \quad \mathbf{x}_x = \begin{Bmatrix} X_{coil} \\ T_x \end{Bmatrix} \quad \mathbf{x}_y = \begin{Bmatrix} X_{coil} \\ T_y \end{Bmatrix}$$

The vectors $\boldsymbol{\Delta}_m$, $\boldsymbol{\Delta}_{mx\tau}$ and $\boldsymbol{\Delta}_{my\tau}$ represent the mass parameters:

$$\boldsymbol{\Delta}_m = \begin{Bmatrix} \Delta_{mx} \\ \Delta_{my} \end{Bmatrix} \quad \boldsymbol{\Delta}_{mx\tau} = \begin{Bmatrix} \Delta_{mx\tau} \\ \Delta_{\tau}^m \end{Bmatrix} \quad \boldsymbol{\Delta}_{my\tau} = \begin{Bmatrix} \Delta_{my\tau} \\ \Delta_{\tau}^m \end{Bmatrix}$$

and the vectors $\boldsymbol{\Delta}_k$, $\boldsymbol{\Delta}_{kx\tau}$ and $\boldsymbol{\Delta}_{ky\tau}$ contain the stiffness parameters:

$$\Delta_k = \begin{Bmatrix} \Delta_{kx} \\ \Delta_{ky} \end{Bmatrix} \quad \Delta_{kx\tau} = \begin{Bmatrix} \Delta_{kx} \\ \Delta_{k\tau}^k \end{Bmatrix} \quad \Delta_{ky\tau} = \begin{Bmatrix} \Delta_{ky} \\ \Delta_{k\tau}^k \end{Bmatrix}$$

A schematic representation of the generation of the transversal displacement $X_{coil}(s)$ is shown as a signal flow chart in Figure 7.

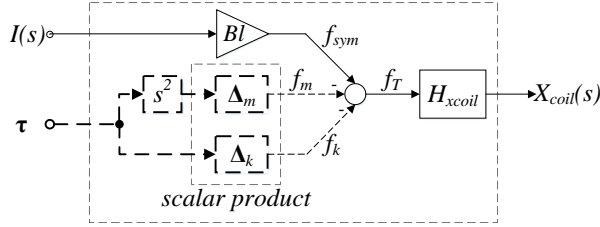


Figure 7 Generation of forces producing voice coil displacement due to coupling with tilting angles

The mechanism is a feedback structure, which requires the prior computation of the tilting angles τ . Physically the coupling forces f_m and f_k are dependent on the amount of tilting of the diaphragm. However, the forces f_m and f_k caused by the rocking modes are usually much smaller than the symmetrical force f_{sym} and can be neglected for the modeling of the voice coil displacement $X_{coil}(s)$.

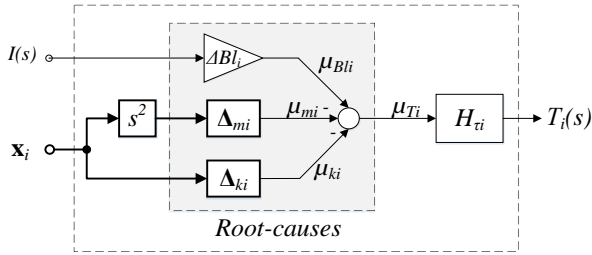


Figure 8 Generation of tilting angles due to moments produced by mass, stiffness and Bl problems

The signal flow chart in Figure 8 shows the generation of the tilting angles $T_i(s)$. The moments μ_{mi} and μ_{ki} representing mass and stiffness excitations depends on the states \mathbf{x}_i , dominated by X_{coil} . The results obtained in Equation (18), and Equation (19) have a high diagnostic value since the root-causes of the rocking modes are clearly separated allowing an intuitive understanding of the problems presents in the loudspeakers and a clear interpretation of the parameters. This brings benefits for the system identification process which will be extended in the part 2 of the present paper.

3.4. Simplified Model

By applying the assumption that the feedback sources due to mass and stiffness problems in Figure 8 are

dominated by X_{coil} on the vector \mathbf{x}_i , and that the tilting angles τ are sufficiently small in Figure 7, the model reduces to the following feed forward structure:

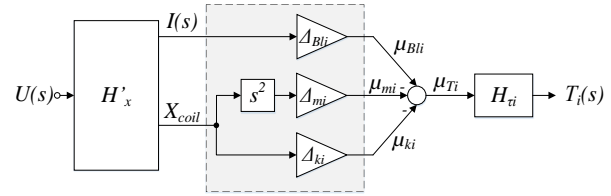


Figure 9 Simplified feed forward model generating the tilting angle $T_i(s)$ of the rocking mode

Note that the block H'_x produces the voice coil displacement and the current before distributing them in the network in charge of producing the moments that are going to excite the tilting resonators, it contains implicitly H_{xcoil} and Z_e^{-1} . Due to its feed forward structure, this model is a much more powerful tool for analysis and diagnostics.

Working in the x - y system of coordinates simplifies some steps for the modelling and for the system identification but is not suitable for analysis and data extraction. For this reason, the moments and the tilting produced by the root-causes need to be expressed in a coordinate system more related to the physics of the loudspeaker vibration. A more convenient alternative is the modal basis.

4. MODAL ANALYSIS

At low frequencies the diaphragm vibration can be described by the superposition of three natural modes; the piston and the two orthogonal rocking modes. From the dynamic analysis the mode shapes and resonance frequencies of the structure depend only on the elasticity and inertial properties [8].

The modal analysis is a powerful tool which allows to analyse complex vibrating systems as a set of simple harmonic oscillators with characteristics vibration patterns called Normal Modes [7]. In the analysis of rocking modes this is a convenient way to decouple the motion equation obtaining directly the rocking resonators and the mode shapes.

4.1. Rocking mode resonators $H_{\alpha 1,2}$

The first step to perform a modal analysis is to find the set of orthonormal basis in which the motion equation will be projected. Some drivers due to its suspension parts or particular acoustic loads present viscoelastic effects [9], [10] that limits slightly the performance of the

modal analysis, since there is not simple way to decouple a system of equations with a frequency dependent stiffness matrix. In this context the viscoelastic terms will be neglected.

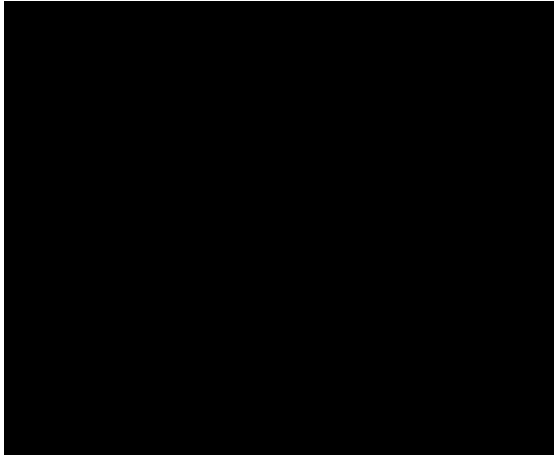
Assuming that there are no forces or moments acting on the loudspeaker and considering only the inertia and elasticity terms, the following eigenvalue problem is posed:

$$(\mathbf{M}^{-1}\mathbf{K} - \omega_n^2\mathbf{I})\Phi_n = \mathbf{0} \quad (20)$$

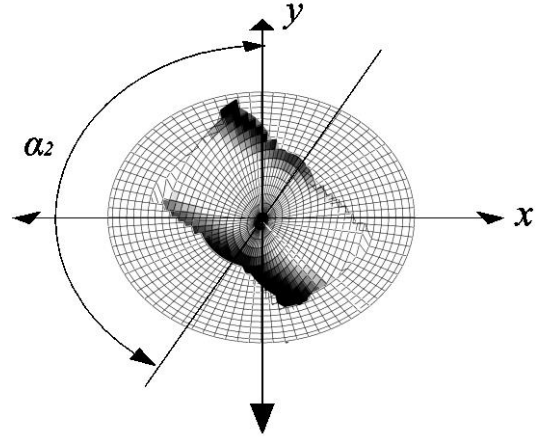
Where the system eigenvalues ω_n^2 corresponds with the resonance frequencies of the fundamental mode f_s and the two orthogonal rocking modes $f_{\alpha 1,2}$ in Hz. \mathbf{I} states for the identity matrix.



Piston Mode Φ_x



Dominant Rocking Mode $\Phi_{\alpha 1}$



Orthogonal Rocking Mode $\Phi_{\alpha 2}$

Figure 10 Mode shapes of a rectangular loudspeaker

The eigenvectors Φ_n are normalized with respect to the mass matrix and correspond to the mode shapes of the fundamental mode Φ_x (piston mode) and the rocking modes $\Phi_{\alpha 1}$ and $\Phi_{\alpha 2}$ as depicted in Figure 10.

The eigenvectors Φ_n are orthogonal to each other and comprise three elements:

$$\Phi_x = \begin{Bmatrix} x_{coil} \\ \approx 0 \\ \approx 0 \end{Bmatrix} \quad \Phi_{\alpha 1} = \begin{Bmatrix} \approx 0 \\ \tau_{x1} \\ \tau_{y1} \end{Bmatrix} \quad \Phi_{\alpha 2} = \begin{Bmatrix} \approx 0 \\ \tau_{x2} \\ \tau_{y2} \end{Bmatrix}$$

The tilting components τ_{xi} and τ_{yi} of the first and second rocking mode ($i=1,2$) describe the direction of the

$$\alpha_i = \arctan\left(\frac{\tau_{xi}}{\tau_{yi}}\right) \quad i = 1,2 \quad (21)$$

maximum tilting in the xy plane as shown in Figure 11.

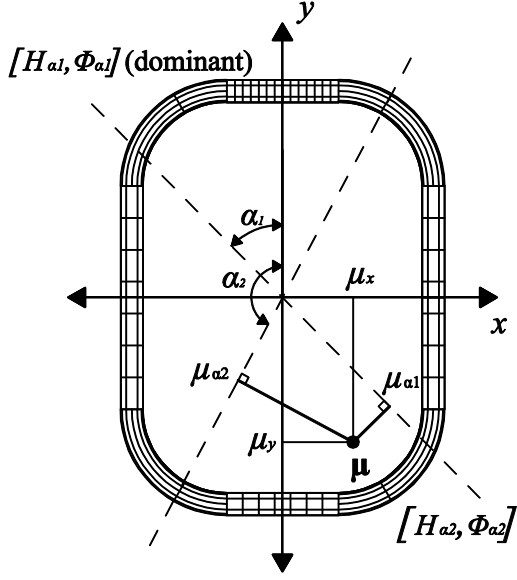


Figure 11 Rotation axes and moment projections

Exploiting the orthogonality between the Eigenvectors the state vector \mathbf{x} in Eq. (8) can be written as a modal expansion

$$\mathbf{x} = \Phi \mathbf{H}(s) \quad (22)$$

with the modal matrix

$$\Phi = [\Phi_x \ \Phi_{\alpha 1} \ \Phi_{\alpha 2}] \quad (23)$$

comprising the orthogonal Eigenvectors Φ_n (mode shapes) and the transfer function matrix:

$$\mathbf{H}(s) = [H_x(s) \ H_{\alpha 1}(s) \ H_{\alpha 2}(s)]^T \quad (24)$$

comprising the mechanical transfer functions:

$$H_x(s) = \frac{1}{s^2 + 2\eta_0\omega_0s + \omega_0^2} \quad (25)$$

corresponding to the piston mode of the diaphragm and the transfer function of the two rocking modes:

$$H_{\alpha n}(s) = \frac{1}{s^2 + 2\eta_{\alpha n}\omega_{\alpha n}s + \omega_{\alpha n}^2} \quad n = 1, 2 \quad (26)$$

shown in Figure 12. Since the mode shapes are normalized with respect to the mass matrix, the rocking resonators are totally described by only two parameters, the resonance frequency and the damping factor [11].

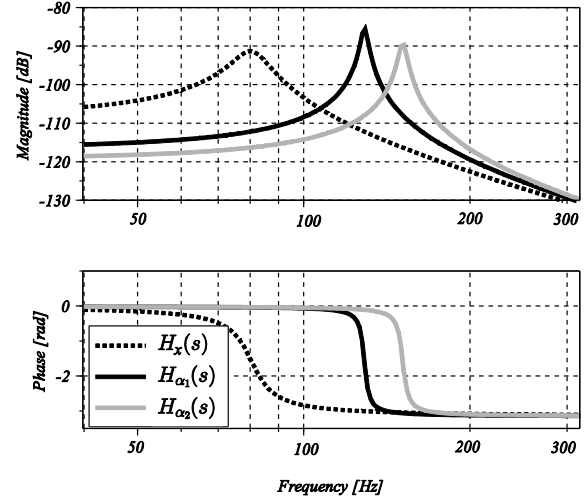


Figure 12 The amplitude and phase response of the modal transfer functions $H_x(s)$, $H_{\alpha 1}(s)$ and $H_{\alpha 2}(s)$

The resonance frequency $\omega_0 = 2\pi f_0$ is the fundamental resonance of the driver and $\omega_{\alpha n}$ with $n=1,2$ are the resonance frequencies of the rocking modes. The modal damping factors η_0 and $\eta_{\alpha n}$ are the mechanical losses of the modes excluding the electrical damping. Theoretically there are two rocking modes but in practice often only one of them is highly excited masking the effect of the other or they degenerate and can be seen as only one with one resonance frequency [12].

4.2. Moments exciting rocking modes

The total moment μ_T exciting the rocking modes

$$\begin{aligned} \mu_T &= \mu_{Bl} - \mu_m - \mu_k \\ \begin{Bmatrix} \mu_{Tx} \\ \mu_{Ty} \end{Bmatrix} &= \begin{Bmatrix} \mu_{Blx} \\ \mu_{Bly} \end{Bmatrix} - \begin{Bmatrix} \mu_{mx} \\ \mu_{my} \end{Bmatrix} - \begin{Bmatrix} \mu_{kx} \\ \mu_{ky} \end{Bmatrix} \end{aligned} \quad (27)$$

and the components μ_{Bl} , μ_m and μ_k corresponding to the root causes mass, stiffness and force factor in Eq. (18) and (19) in Cartesian coordinates are transformed into the modal space to simplify the interpretation:

$$\begin{aligned} \mu_{T\alpha} &= \mu_{Bl\alpha} - \mu_{m\alpha} - \mu_{k\alpha} \\ \begin{Bmatrix} \mu_{T\alpha 1} \\ \mu_{T\alpha 2} \end{Bmatrix} &= \begin{Bmatrix} \mu_{Bl\alpha 1} \\ \mu_{Bl\alpha 2} \end{Bmatrix} - \begin{Bmatrix} \mu_{m\alpha 1} \\ \mu_{m\alpha 2} \end{Bmatrix} - \begin{Bmatrix} \mu_{k\alpha 1} \\ \mu_{k\alpha 2} \end{Bmatrix} \\ &= \mathbf{R}\mu_T \\ &= \mathbf{R}(\mu_{Bl} - \mu_m - \mu_k) \end{aligned} \quad (28)$$

where the total moment $\mu_{T\alpha}$ and its components are aligned with the direction of the modes by using the projection matrix:

$$\mathbf{R} = \begin{bmatrix} -\sin \alpha_1 & \cos \alpha_1 \\ -\sin \alpha_2 & \cos \alpha_2 \end{bmatrix} \quad (29)$$

based on the angles defined in Eq. (21) and shown in Figure 11. The signal flow chart in Figure 13 illustrates the modeling of the rocking modes in the model space.

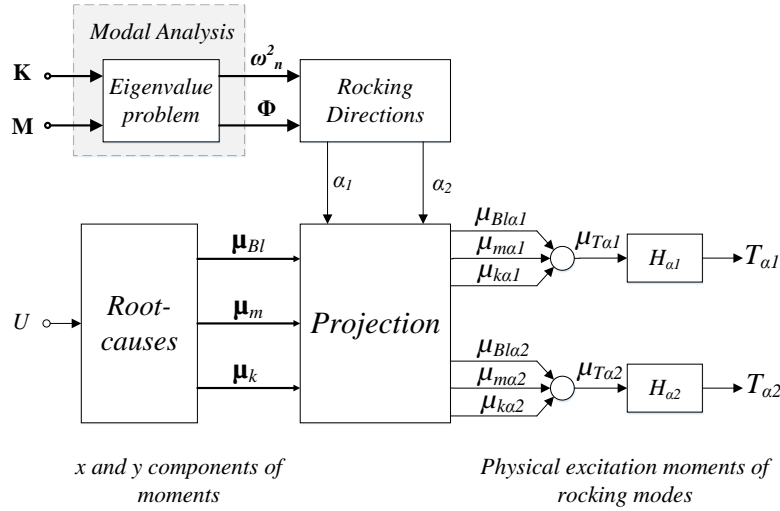


Figure 13 Generation of total moments activating rocking modes

5. DISCUSSION

The proposed model is validated by conducting several experiments related with the three root causes that need to be identified and separated in practical diagnostics. The idea is to intentionally excite the rocking modes of the loudspeaker by modifying its mechanical and magnetic properties and to quantify its effects by means of the identification of the root-causes moments. The system identification scheme and the interpretation of the estimated parameters will be discussed in greater detail in part 2.

For instance, to produce a substantial tilting angle of the diaphragm due to a mass unbalances, the center of gravity of the moving parts is shifted by placing a small mass on the surface. In the other hand, a stiffness asymmetry was created by perforating the surround along a determined area without changing the amount of mass displaced. The B-field inhomogeneity was generated by gluing two arrays of neodymium magnets with inverse polarity in opposite directions to the back plate. More details on the experimental setup will be given in the second part of this paper.

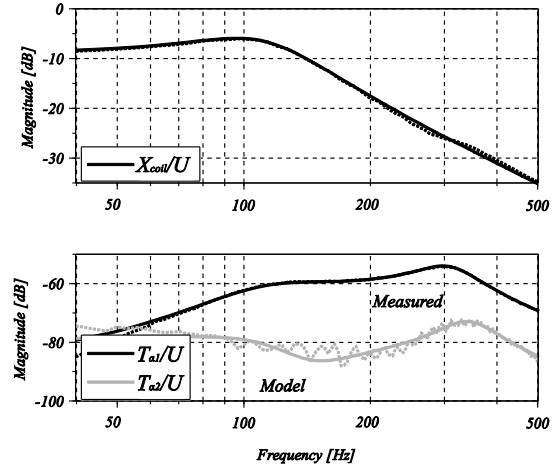


Figure 14 Amplitude response of the fundamental and two rocking modes measured (dotted) and modeled (solid) on a transducer perturbed by an additional mass shifting the center of gravity.

5.1. Dominant mass problem

The experimental results and the fitted model of the rocking behavior due to a mass perturbation are shown in Figure 14.

The moment μ_{ma1} exciting the first mode generated by the additional mass is proportional to the acceleration which is the 2nd derivative of the transversal displacement X_{coil} (see Figure 9) but the tilting angle T_{a1} shown in Figure 14 reveals an additional resonance peak at 300 Hz and an additional decay 12 dB per octave at higher frequency caused by the modal resonator with the transfer function $H_{a1}(s)$.

Note that the amplitude response of the tilting angle T_{a2} of the second mode reveals other root causes activating the second resonator $H_{a2}(s)$ having a resonance frequency at 350 Hz. The output parameters and moments computed in the identified system (solid line) will be explained in the second part of the paper.

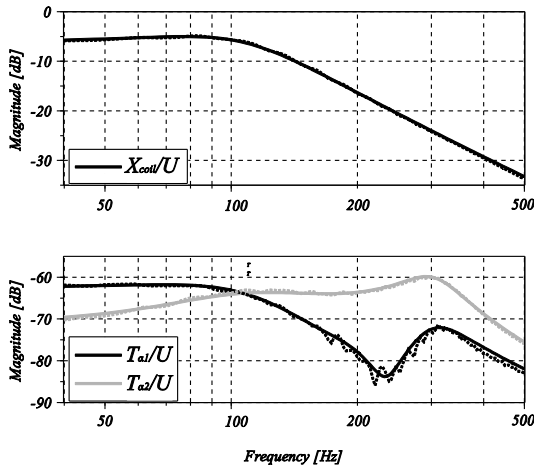


Figure 15 Amplitude response of the fundamental and two rocking modes measured (dotted) and modeled (solid) on a transducer perturbed by an asymmetrical distribution of the circumferential stiffness.

5.2. Dominant stiffness problem

The measured and modeled amplitude responses of the transversal displacement X_{coil} and the tilting angles of the three modes of a transducer perturbed by an asymmetrical stiffness distribution are shown in Figure 15. The moment μ_{ka1} exciting the first mode generated by the stiffness is in accordance to Figure 9 proportional to the voice coil displacement X_{coil} generating the low pass

characteristic of the tilting angle $T_{a1}(s)$. The modal resonator with the transfer function $H_{a1}(s)$ generates the peak at 320 Hz and decay at higher frequencies. The amplitude response $T_{a2}(s)$ of the second mode reveals a mass unbalance as found in the original transducer without any perturbation.

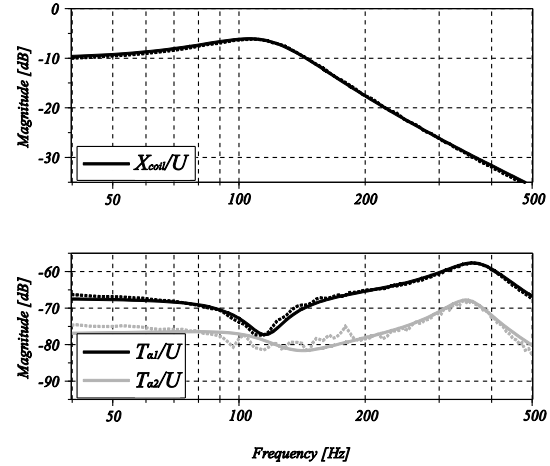


Figure 16 Amplitude response of the fundamental and two rocking modes measured (dotted) and modeled (solid) on a transducer perturbed by an asymmetrical force factor distribution in the magnetic gap.

5.3. Dominant BI problem

Figure 16 shows the amplitude responses of the first three modes of a transducer with an asymmetrical Bl distribution in the gap. Contrary to the root causes associated to mass and stiffness, the moment $\mu_{Bl a1}$ generated by the force factor asymmetry has a relative flat response corresponding to the spectrum of the electrical current $I(s)$ used in the excitation term in Figure 9. The impedance maximum fundamental resonance frequency $f_0 = 120$ Hz causes a dip in the amplitude response of the tilting angle $T_{a1}(s)$. The following modal resonator with the transfer function $H_{a1}(s)$ generates the resonance peak and the decay above 300 Hz. The frequency response of the 2nd rocking mode $T_{a2}(s)$ reveals a small stiffness asymmetry found in the original transducer.

6. CONCLUSIONS

A new model has been presented describing the generation of the fundamental mode and the first two rocking modes. The quantification of the moments

exciting the modal resonators reveal the root causes of the rocking modes which are caused by asymmetrical distributions of mass in the moving elements, stiffness in the suspension and electrodynamic force factor.

The particular characteristics of each problem has been explained from the physical and from the system oriented modelling approach clarifying the symptoms associated to each problem.

The transfer responses associated to mass, stiffness and *Bl* problems are discussed and the differences between the associated symptoms have been clearly separated. The model has been experimentally validated by the perturbation analysis of the three causes.

Numerical modeling based on the theory developed in this paper is the basis for simulating the mechanical vibration but also for the root cause analysis of rocking modes and other kinds of diagnostics. The identification of the free model parameters based on measured data provided by laser scanning are the basis for performing practical diagnostics on a particular transducer.

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