Nonlinear Adaptive Controller for Loudspeakers with Current Sensor

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Abstract:

Nonlinear loudspeaker distortions are predictable and can be compensated by inverse preprocessing of the electric input signal. Control schemes developed so far require an acoustic or mechanic output signal measured at the speaker. An additional sensor increases the costs and is impractical under harsh environment. However, a motional signal can also be derived from the back-induced EMF by monitoring the input current of the transducer only. Following this approach a novel nonlinear adaptive detector system is presented providing a robust and cost-effective DSP solution.

1. Introduction

Nonlinearities inherent in common loudspeakers produce new spectral components (distortion) in the reproduced sound which effect the perceived sound quality and impair active noise and vibration control. Recent research has shown that these nonlinear mechanisms are predictable and a nonlinear control can compensate for these distortions by inverse preprocessing of the electric input signal. Adaptive schemes have been developed to adjust the control parameters to the particular transducer and to cope with parameter uncertainties due to heating and aging.

A self-tuning control system requires a measured input signal which is closely related with the loudspeaker output signal and describes its performance. Table 1 gives a summary on possible monitored signals, possible sensors, pro and cons in practical application.

The measurement of the sound pressure signal by a microphone is the most direct way. In active attenuation systems for air borne noise microphones are already in use for noise cancellation and can also used for loudspeaker linearization. However, these signals have a time delay, are corrupted by primary noise and influenced by the acoustical environment. The measurement without mechanical contact is a clear advantage but can also be accomplished by optical or capacitive sensors. Optical displacement meters (Laser) are expensive and restricted to research and development only. Also the price of a high-quality capacitive sensor exceeds the costs of the loudspeaker system in most cases. Inexpensive accelerometers are available but their attachment to the moving diaphragm produces mechanical and practical problems. The measurement of the velocity with an additional sensing coil requires a special loudspeaker design resulting in higher moving mass, less efficiency and higher costs.

However, electrodynamic loudspeakers can be used as sensor itself while reproducing an audio signal at the same time. The back electromotive force (EMF) produces a direct relationship between the electric input signals at the loudspeaker's terminals and the velocity of the voice-coil. The control system shown in Fig. 1 requires only the measurement of the current and voltage.

The idea is not new [1], [2] but the detection of the loudspeaker information in the required accuracy has not yet been solved by analogue techniques. Digital signal processing opens new ways for applying advanced system identification techniques and for coping with loudspeaker nonlinearities. After discussing the known techniques and their limiting factors the paper defines the requirements and presents the algorithm for a self-tuning detector system. Finally the paper discusses the implementation in known control architectures (servo control, state feedback, mirror filter).

2. Loudspeaker Model

The basis for designing a detector system are the results of loudspeaker modeling [6] describing the low-frequency behavior of loudspeakers at large amplitudes. Fig. 2 shows a nonlinear equivalent circuit comprising constant lumped elements

R_e dc resistance of driver voice-coil,
m mechanical mass of driver diaphragm assembly including voice-coil and air load,
R_m mechanical resistance of total-driver losses,
$Z_A(s)$ impedance of the acoustic system (radiation aid)

and the nonlinear elements which are instantaneous force factor

$$b(x) = \sum_{j=0}^{N} b_j x^j \tag{1}$$

instantaneous stiffness of the driver suspension

$$k(x) = \sum_{j=0}^{N} k_j x^j \tag{2}$$

and instantaneous inductance of driver voice-coil

$$L_e(x) = \sum_{j=0}^{N} l_j x^j \tag{3}$$

expanded into a power series and using the quantities (time signals)

x(t) voice-coil displacement,
v(t)=dx/dt velocity,
u(t) voltage at the loudspeaker terminals,
i(t) input current and
$F_m(t)=(dL(x)/dx)i^2/2$ the electromagnetic driving force.

3. Linear Detector

There are early attempts like those described by De Boer [3] for sensing cone velocity by placing the driver in a bridge arrangement and using the detected signal in servo control. Other [4], [5] used current feedback and filtering in the forward transfer function of the power amplifier. Again, a feedback signal is obtained that is proportional to the voice coil velocity times the instantaneous force factor. The transformation is based on an assumed linear relationship

$$v_E = \frac{1}{b_0}\left(u_L - R_e i_L - l_0 \frac{di_L}{dt}\right) \tag{4}$$

between estimated velocity v_E and measured voltage u_L and current i_L. A controller with such a linear detector allows modification of the linear transfer response as long as the loudspeaker nonlinearities are sufficiently small. However, this concept fails in the compensation of nonlinear loudspeaker distortion as shown in the following simulation.

3.1 Servo Control with a Linear Detector

A loudspeaker with servo control using a feedback of voice-coil velocity sensed by a linear detector circuit is modeled by Fig. 3. The nonlinear loudspeaker is represented by a linear system $q_x(s)$ describing the electromechanical conversion, a second linear system with the transfer function $q_r(s)$ corresponding with the radiation and sound propagation to the listening point and two nonlinear subsystems. The nonlinear subsystems produce the distortion in the output signal and the nonlinear input current I(s).

For the further discussion of the behavior of the controller we combine the linear detector with the nonlinear loudspeaker and use a modified model presented in Fig. 4. The estimation of the sensed velocity $V_E(s)$ is described by a parallel connection of a simple differentiator and a nonlinear system which adds additional distortion to the true velocity V(s).

The controller comprises two linear filters $H_C(s)$ and $H_P(s)$. The filter in the negative feedback path $H_C(s)$ realizes a desired transfer function in the open loop

$$K(s) = s q_x(s) H_C(s) \tag{5}$$

where s is the Laplace operator and

$$q_x(s) = \frac{X(s)}{U_L(s)} = \frac{b(0)}{(R_e + L_e s)\left(ms^2 + R_m s + k(0) + Z_A(s)s\right) + b(0)^2 s} \tag{6}$$

represents the linear transfer function between voltage $U_L(s)$ at the loudspeaker terminals and voice-coil displacement X(s). Since we focus in this simulation on the distortion reduction a second filter with the open loop transfer function $H_P(s)=1+K(s)$ is connected to the control input to preserve the linear properties of the loudspeaker (e.g. resonance frequency and loss factor) in the transfer response of the overall system.

Using the Volterra-series approach the nonlinear transfer behavior between control input U(s) and sound pressure P(s) can be approximated by a parallel connection of homogeneous power systems (pure linear, quadratic, cubic subsystems) as shown in Fig. 5. This approximation is valid as long as the amplitude of the fundamental $p_{lin}(t)$ generated by the first-order system function

$$G_1(s) = H_1(s) = q_x(s) q_r(s) \tag{7}$$

is large in comparison to the amplitude of nonlinear distortion $p_{nlin}(t)$ produced by the second-order system function

$$G_2(s_1,s_2)=\frac{H_2(s_1,s_2)}{1+K(s_1+s_2)}-\frac{D_2(s_1,s_2)K(s_1+s_2)}{1+K(s_1+s_2)} \tag{8}$$

and the third-order system function

$$G_3(s_1,s_2,s_3)=\frac{H_3(s_1,s_2,s_3)}{1+K(s_1+s_2+s_3)}-\frac{D_3(s_1,s_2,s_3)K(s_1+s_2+s_3)}{1+K(s_1+s_2+s_3)} \tag{9}$$

where $H_1(s)$ is the linear and $H_2(s_1, s_2)$ and $H_3(s_1, s_2, s_3)$ are the higher-order system functions derived for a closed- and vented-box systems in [Eqs. 13-15, 7].

Using a linear detector according Eq. (4) which is unable to compensate for speaker's nonlinearities the second- and third-order system functions in Eqs. (8) and (9) contain an additional term

$$D_2(s_1,s_2)=q_r(s_1+s_2)\Big[\ b_1q_x(s_1)q_x(s_2)/2 \\ +l_1\big(q_x(s_1)q_I(s_2)+q_x(s_2)q_I(s_1)\big)/2+\dots\ \Big] \tag{10}$$

and

$$D_3(s_1,s_2,s_3)=q_r(s_1+s_2+s_3)\Big[\ b_2q_x(s_1)q_x(s_2)q_x(s_3)/3 \\ +l_2\big(q_x(s_1)q_i(s_2)q_x(s_3)+q_x(s_2)q_i(s_1)q_x(s_3)+q_x(s_1)q_i(s_2)q_x(s_3)\big)/3 \\ +\dots\ \Big] \tag{11}$$

respectively, depending on the nonlinear coefficients of the power series expansion in Eqs. (1) and (3) and the linear transfer function

$$q_i(s)=\frac{I_L(s)}{U_L(s)}=\frac{1-b(0)sq_x(s)}{R_e+L_es}. \tag{12}$$

As shown in previous works [6] the higher-order system functions are a convenient basis for predicting the amplitude response of harmonic and intermodulation distortion. Assuming a loudspeaker with asymmetric force factor characteristic ($b_1\neq 0$) having a resonance frequency f_r=50 Hz and a total loss factor of Q=1.0 we can investigate the effect of servo control in respect with distortion reduction. Fig. 6 presents the amplitude of the harmonic distortion components at 2f as the function of the fundamental tone f. The solid line represents the loudspeaker without servo control where the control filter $H_C(s)$=C=0. By increasing the gain C the nonlinear distortion can only marginally reduced below the resonance frequency but the distortion reduction fails at higher frequencies. Whereas the amplitude of harmonic distortion decreases with growing frequency, force factor nonlinearities generate also broad-band intermodulation components of high amplitude as discussed in [7]. Fig. 7 shows the summed-tone intermodulation generated by a loudspeaker excited with a two-tone signal comprising a first tone with variable frequency f_1 used as abscissa and a second tone at the resonance frequency $f_2=f_r$. Again the solid line shows the distortion of the loudspeaker without feedback ($H_C(s)=C=0$). Increasing the feedback gain C even increases the distortions in the interested frequency range. The poor performance is caused by the nonlinear distortion in the monitored input current generating the second term in Eqs. (8) and (9) which does not vanish for high feedback gain C in K(s).

4. Nonlinear Detector System

4.1 Requirements

Summarizing the results of the previous simulation the requirements on a precise detector system can be defined as follows:

- The detector circuit should be based on a physical model of the loudspeaker. That results in highest accuracy possible while introducing a minimal number of unknown parameters which have to be identified.
- The detector should consider the speaker's nonlinearities.
- The detector should be adaptive to make the identification of the unknown parameters feasible while reproducing an audio signal.
- The adaptive detector algorithm should be stable and robust under all conditions.

4.2 BASIC EQUATIONS

There are two different detector circuits possible depending on the physical parameters and signals involved.

4.2.1 Detector Based on Voltage Equation

Using the relationship on the electric side of the equivalent circuit in Fig. 2 the velocity

$$\frac{dx}{dt} = \frac{1}{b(x)}\left(u_L - R_e i_L - \frac{d\left(L_e(x) i_L\right)}{dt} \right) \tag{13}$$

can be calculated from voltage $u_L(t)$, current $i_L(t)$ and using particular loudspeaker parameters. The nonlinear parameters require the displacement x generated from the integrated velocity forming a feedback loop as shown in Fig. 8. The detector is independent of the mechanical and acoustical loudspeaker parameters.

4.2.2 Detector Based on Force Equation

The relationship in the mechanical part of the equivalent circuit can be used for estimating the voice coil velocity

$$\frac{dx}{dt} = L^{-1}\left\{\frac{s}{J(s)}\right\} * \left[b(x)i_L - [k(x)-k(0)]x + \frac{i_L^2}{2}\frac{dL(x)}{dx}\right] \tag{14}$$

from the input current $i_L(t)$ where $L^{-1}\{\}$ is the inverse Laplace transformation, * is the convolution and the transfer function

$$J(s) = ms^2 + R_m s + k(0) + Z_A(s) \tag{15}$$

comprises only linear loudspeaker parameters. The corresponding signal flow chart is presented in Fig. 9 comprising a linear filter $J(s)^{-1}$, a differentiator, three static nonlinearities, adders and multipliers. The detector based on the force equation requires more parameters to be identified than the detector based on the voltage equation.

4.3 Adaptive Parameter Adjustment

The free parameters in the detector have to be identified for the particular loudspeaker. This paper focuses on the estimation of the coefficients in the power series expansion in Eqs. (1 -3). The adjustment of the linear parameters is skipped here since these problems are more straightforward and standard solutions are sufficiently published elsewhere [8] and [9].

4.3.1 Definition of the error signal

Estimation of the free parameters in the detector is essentially an optimization problem where the model (detector) is as closely as possible adjusted to the physical plant (loudspeaker). The disagreement is quantified by an error signal and the adaptive algorithms search for a minimum of the error signal.

A convenient error signal

$$\begin{aligned} e &= \left.\frac{dx}{dt}\right|_{Eq.13} - \left.\frac{dx}{dt}\right|_{Eq.14} \\ &= \frac{1}{b(x)}\left(u_L - R_e i_L - \frac{d(L_e(x)i_L)}{dt}\right) \\ &\quad - L^{-1}\left\{\frac{s}{J(s)}\right\} * \left[b(x)i_L - [k(x)-k(0)]x + \frac{i_L^2}{2}\frac{dL(x)}{dx}\right] \end{aligned} \tag{18}$$

is the difference between the velocity estimated via the voltage Eq. (13) and the velocity estimated by the force Eq. (14).

4.3.2 Parameter Updating

Defining the cost function as the mean squared error

$$MSE \equiv J = E\left[(e(i))^2\right] \tag{19}$$

the optimal filter parameters are determined by searching for the minimum of the cost function where the partial derivatives of the cost function in respect to the free parameters become zero. That can be accomplished by an iterative calculation widely known as least-mean square (LMS) algorithm. Using the power series expansion of Eq. (1) - (3) the nonlinear coefficients of the force factor

$$b_j(n+1) = b_j(n) + \mu\, e \frac{\partial e}{\partial b_j} \tag{20}$$

Stiffness

$$k_j(n+1) = k_j(n) + \mu\, e \frac{\partial e}{\partial k_j} \tag{21}$$

and inductance

$$l_j(n+1) = l_j(n) + \mu\, e \frac{\partial e}{\partial l_j} \tag{22}$$

are updated by adding the product of the error signal and partial derivatives of e in respect to the coefficient scaled by a learning constant μ.

Approximately the partial derivatives can be calculated for the force factor

$$\frac{\partial e}{\partial b_j} \approx -\left(u_L - R_e i_L - \frac{d\left(L(x) i_L\right)}{dt}\right) \frac{x^j}{b(x)^2} - L^{-1}\left\{\frac{s}{J(s)}\right\} * \left[x^j i_L\right] \tag{23}$$

stiffness

$$\frac{\partial e}{\partial k_j} \approx L^{-1}\left\{\frac{s}{J(s)}\right\} * \left(x^{j+1}\right) \tag{24}$$

and inductance

$$\frac{\partial e}{\partial l_j} \approx -\frac{d\left(x^j i_L\right)}{dt} \frac{1}{b(x)} - L^{-1}\left\{\frac{s}{J(s)}\right\} * \left(\frac{j x^{j-1} i_L^2}{2b(0)}\right). \tag{25}$$

5. Implementation in Nonlinear Control Architectures

5.1 Servo-Control with a Nonlinear Detector

The servo control system discussed in the simulation in chapter 3.1 can reduce the distortion in the output signal if the linear detector defined by Eq. 4 is replaced by a nonlinear detector system represented by Eq. (13) or (14). The second-order transfer function

$$G_2(s_1, s_2) = \frac{H_2(s_1, s_2)}{1 + K(s_1 + s_2)} \tag{16}$$

and the third-order function

$$G_3(s_1, s_2, s_3) = \frac{H_3(s_1, s_2, s_3)}{1 + K(s_1 + s_2 + s_3)} \tag{17}$$

of the overall system (loudspeaker with a nonlinear detector) vanish when the gain of the open loop transfer function K(s) goes to infinity. Fig. 10 and 11 show the amplitude response of the harmonic and intermodulation components of the same loudspeaker and excitation signals used in Fig. 6 and 7. A high gain C in the control filter $H_C(s)$ reduces the distortion components at low frequencies. However servo control becomes ineffective in reducing the intermodulation at higher frequencies since the lower voice coil velocity decreases the open loop gain K(s). To ensure stability of the servo controller the falling loop gain of K(s) is desired or even enhanced by a low-pass characteristic of the control filter $H_C(s)$ [10, 11].

The AD and DA conversion required for the digital implementation of the adaptive algorithms produces additional time delay which can be fatal for the stability of the feedback loop. Therefore, classical servo control combined with an adaptive detection is a hybrid solution between analogue and digital domain producing some problems in practical implementation.

However, there are some alternative control techniques available which can be easier implemented and provide perfect linearization of the speaker in theory.

5.2 Nonlinear State Feedback

Regular static state feedback is a well developed technique for controlling nonlinear systems and has been discussed for the linearization of loudspeakers by [12], [13], [14] . Here the nonlinear controller is connected in series to the loudspeaker as shown in Fig. 12 performing an inverse nonlinear signal processing by a nonlinear control law to compensate for the speaker's nonlinearities. The control law is derived from the nonlinear loudspeaker model in state space representation and is implemented as a special algorithm in a digital processing system. The control law has free parameters which has to be adjusted to the particular loudspeaker and requires permanently a displacement, velocity and current signal describing the state of the loudspeaker. All of this information are available in the nonlinear detector system. Both the control law and the nonlinear detector are based on the same physical loudspeaker model to simplify the transfer of loudspeaker information.

The control law and the detector can be easily implemented in a digital signal processor. Only the measurement of current is required since the voltage signal is already available in digital domain as the control output signal. A time delay caused by the AD-converter for the current measurement derogates the performance but does not necessary endanger the stability of the controller.

5.3 Nonadaptive Filter

Using the same control law as in nonlinear state feedback but generating the required state information from the control input signal leads to the mirror filter [15] or related filter concepts (Volterra filter [6]). Here only the loudspeaker parameters are transferred to the filter only as shown in Fig. 13. Since the loudspeaker parameters are almost constant over a period of time the filter can cope with any time delay produced by the detector circuit or the AD-converter for current measurement. It is also possible to disable the detector system and to keep the filter operative with stored parameters.

5.4 Adaptive Filter

Using an adaptive filter as proposed in [16] and [17] the nonlinear detector generates only one signal corresponding with the loudspeaker output signal. Here the detector takes over the functionality of a conventional sensor. As shown in Fig. 14 the detected velocity is compared with a desired signal produced by a linear system $H_{lin}(s)$ and the resulting error signal is used for updating the adaptive filter.

6. Conclusion

Loudspeaker control requires information on the properties or performance of the loudspeaker. For electrodynamical loudspeaker essential information can be extracted from the electric signals at the loudspeakers terminals by using nonlinear system identification techniques. This concept reduces number, complexity and cost of the hardware components. The measurement of the current can be accomplished by a simple shunt or by inexpensive current sensors available for motor control. A low-cost AD converter can be used to transfer the current into the digital domain where loudspeaker control and parameter identification are realized by software algorithms.

7. References

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Signal	Sensor	Advantages	Problems	Application
sound pressure	microphone	no mechanical contact	time delay, radiation, acoustical environment	active sound control
displacement	optical displacement meter (Laser)	no mechanical contact	cost, dust	research and development
velocity	electrodynamic transducer (second coil)	inexpensive	linearity, higher moving mass	special loudspeaker design
acceleration	accelerator	inexpensive	attachment, robustness	active vibration control
input current	resistor	cost, robustness, linearity	EMF detection requires DSP	digital controller in consumer and professional applications

Table 1: Sensing principles used for transducer control

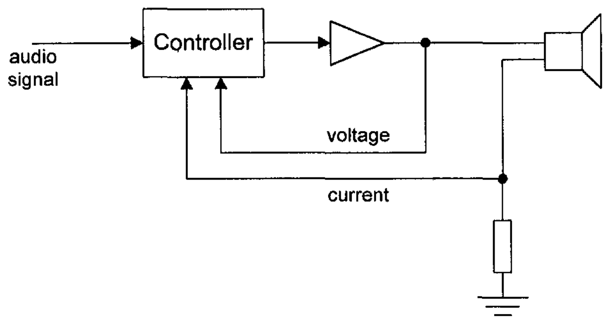

Fig. 1: Loudspeaker controller without additional sensor.

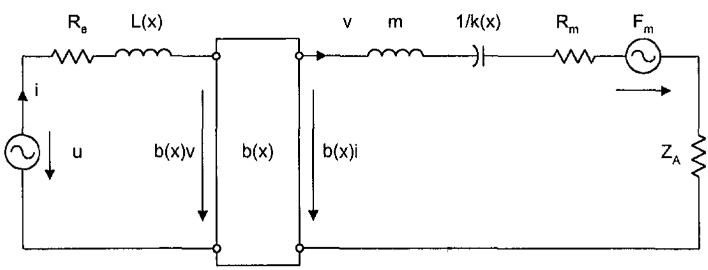

Fig. 2: Equivalent circuit of an electrodynamical transducer.

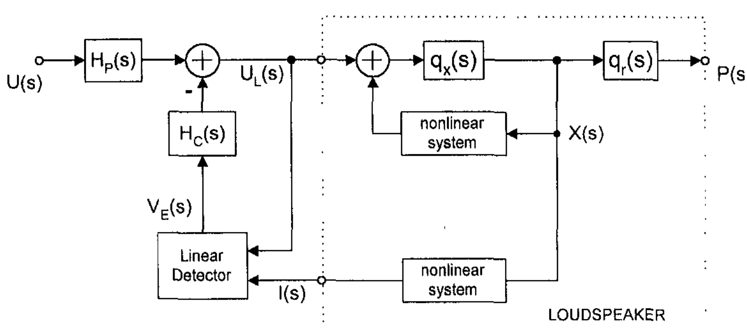

Fig. 3: Servo Control (motional feedback) with a linear detector.

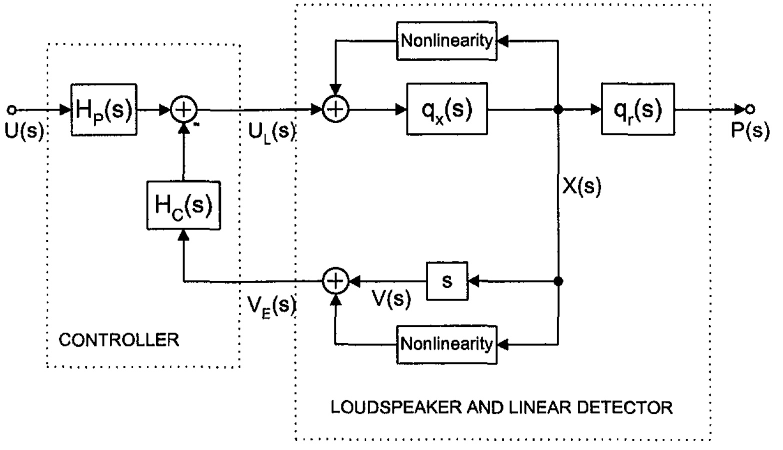

Fig. 4: System model for loudspeaker with linear detector

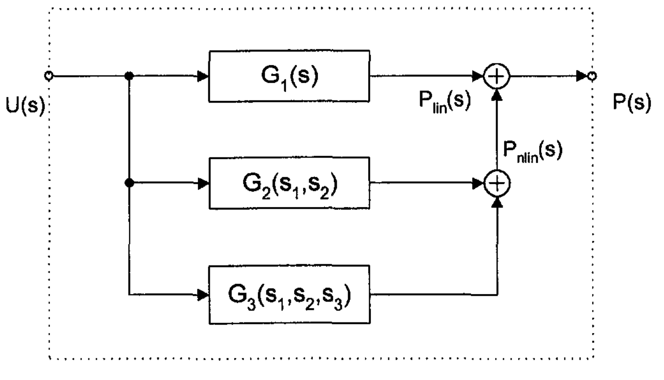

Fig. 5: Nonlinear polynomial system.

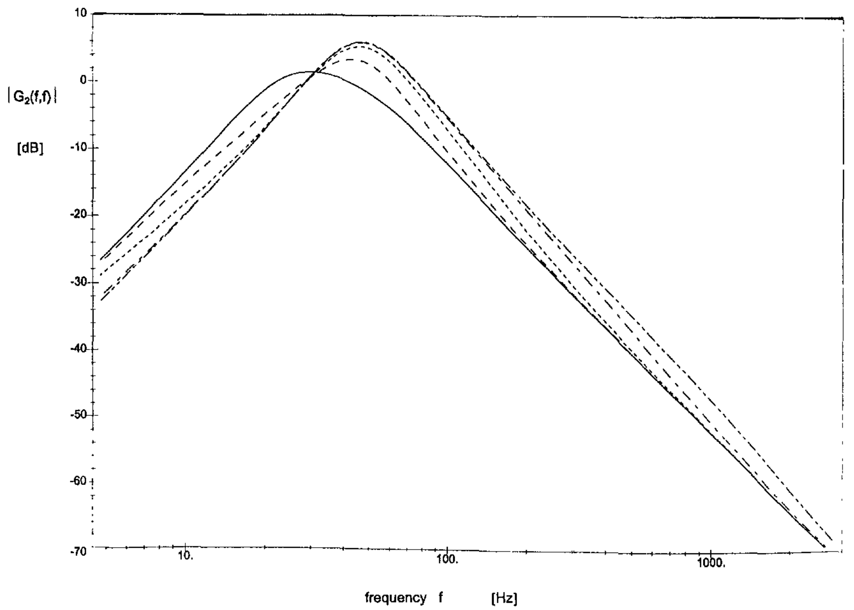

Fig. 6: Second-order harmonic distortion of a loudspeaker with servo control and linear detector.

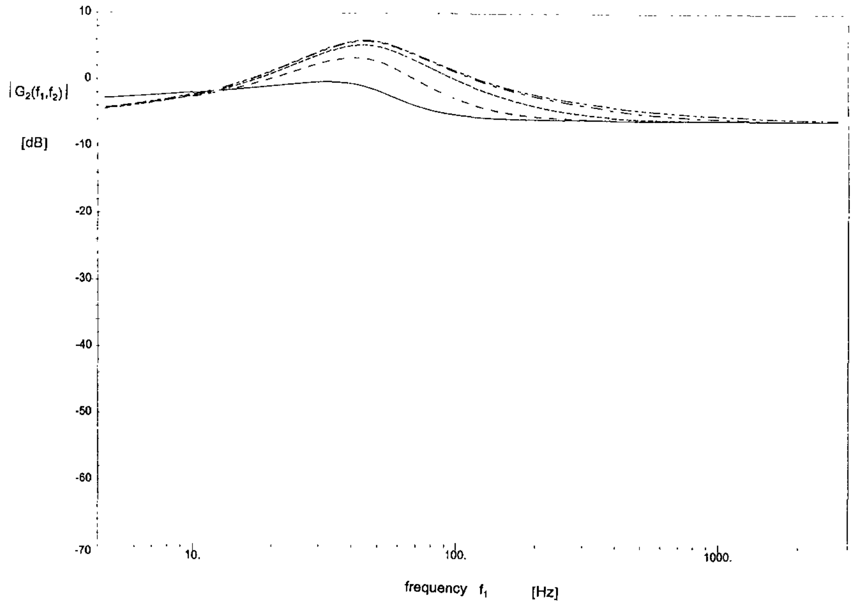

Fig. 7: Second-order intermodulation distortion of a loudspeaker with servo control and linear detector.

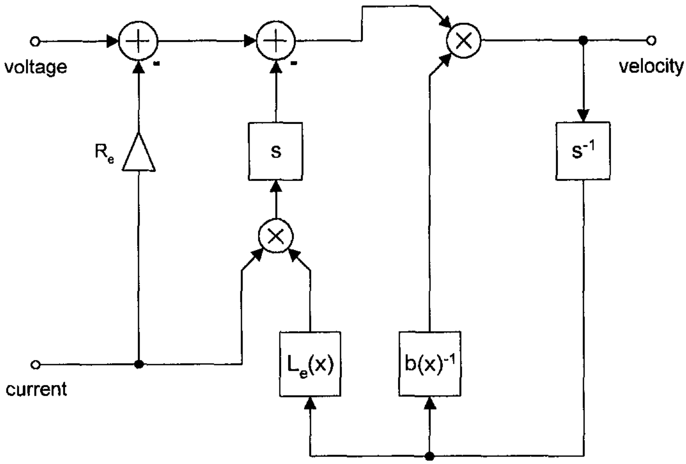

Fig. 8: Nonlinear detector based on voltage equation.

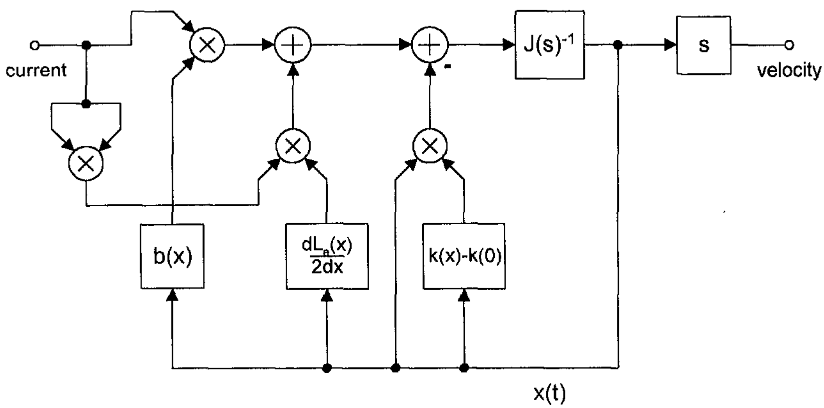

Fig. 9: Nonlinear detector based on mechanical force equation.

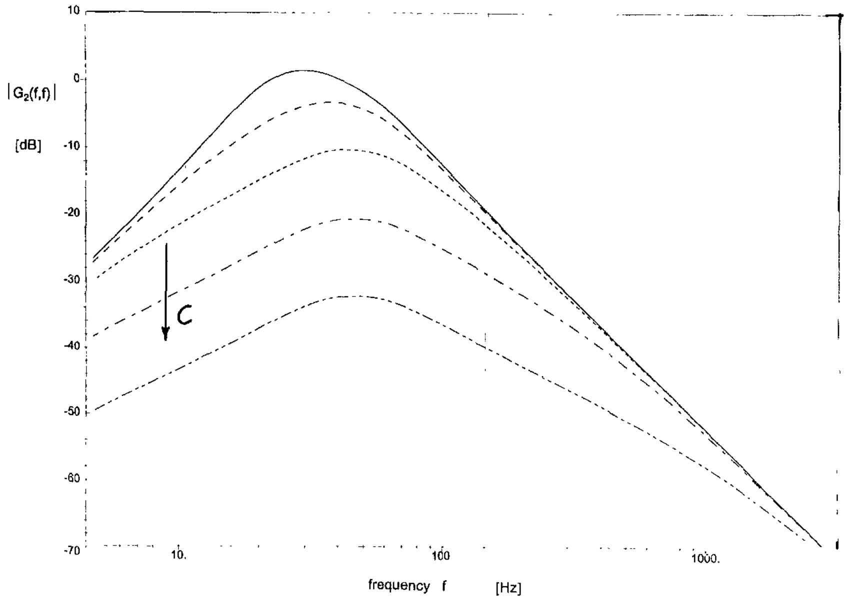

Fig. 10: Second-order harmonic distortion of a loudspeaker with servo control and nonlinear detector.

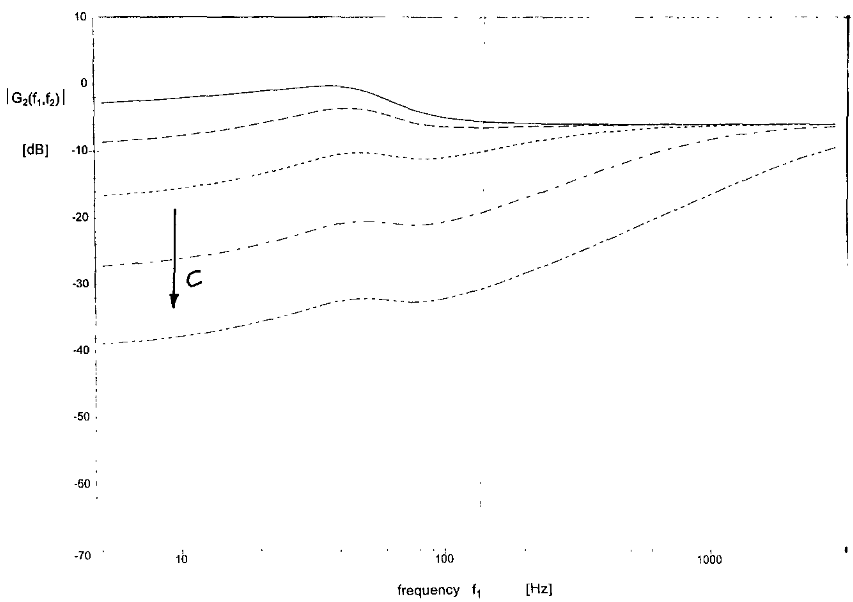

Fig. 11: Second-order intermodulation distortion of a loudspeaker with servo control and nonlinear detector.

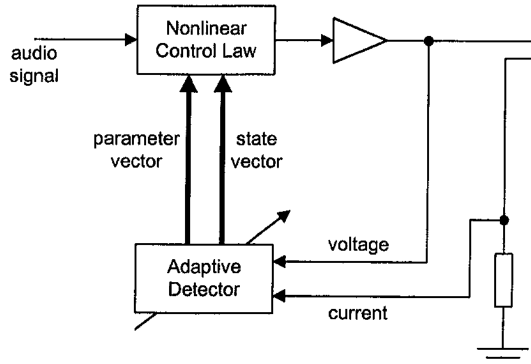

Fig. 12: Nonlinear state feedback control with parameter and state transfer.

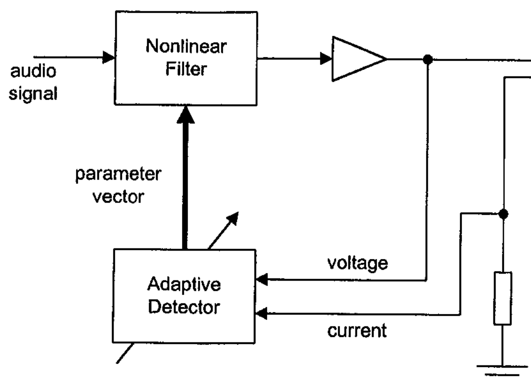

Fig. 13: Loudspeaker control with nonlinear filter and parameter transfer.

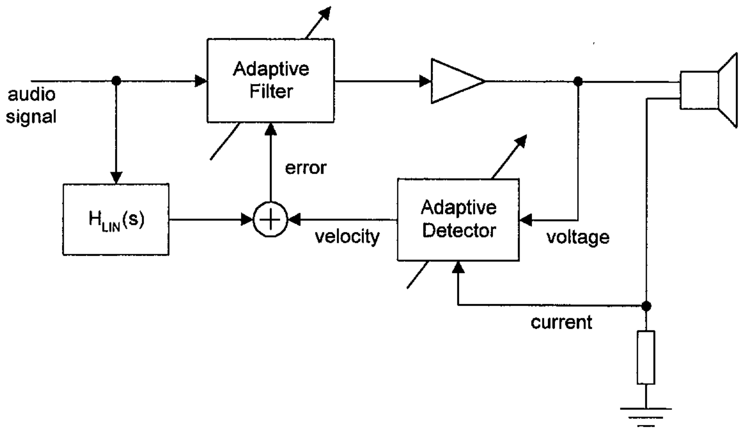

Fig. 14: Loudspeaker control with an adaptive nonlinear filter.