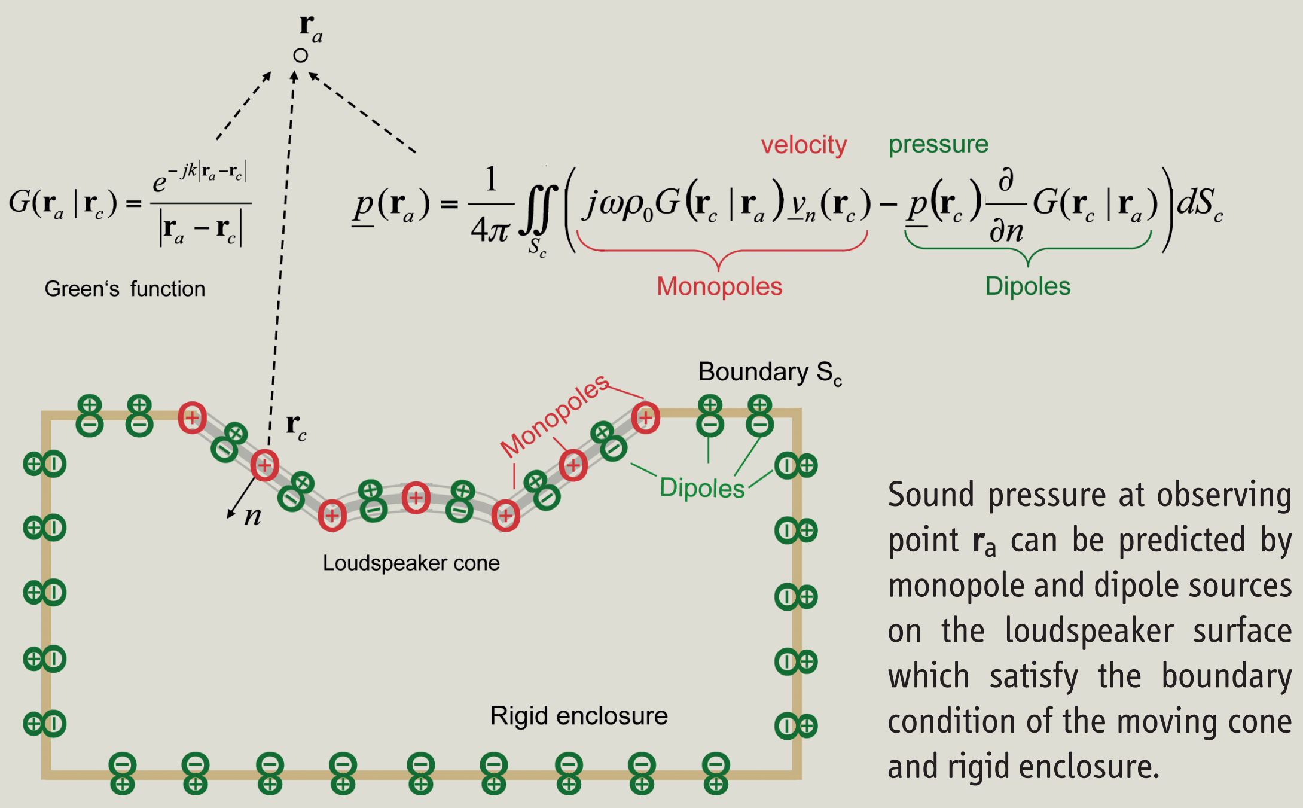
LOUDSPEAKER SOUND RADIATION

FUNDAMENTALS

Boundary Element Method

Kirchhoff-Helmholtz Equation

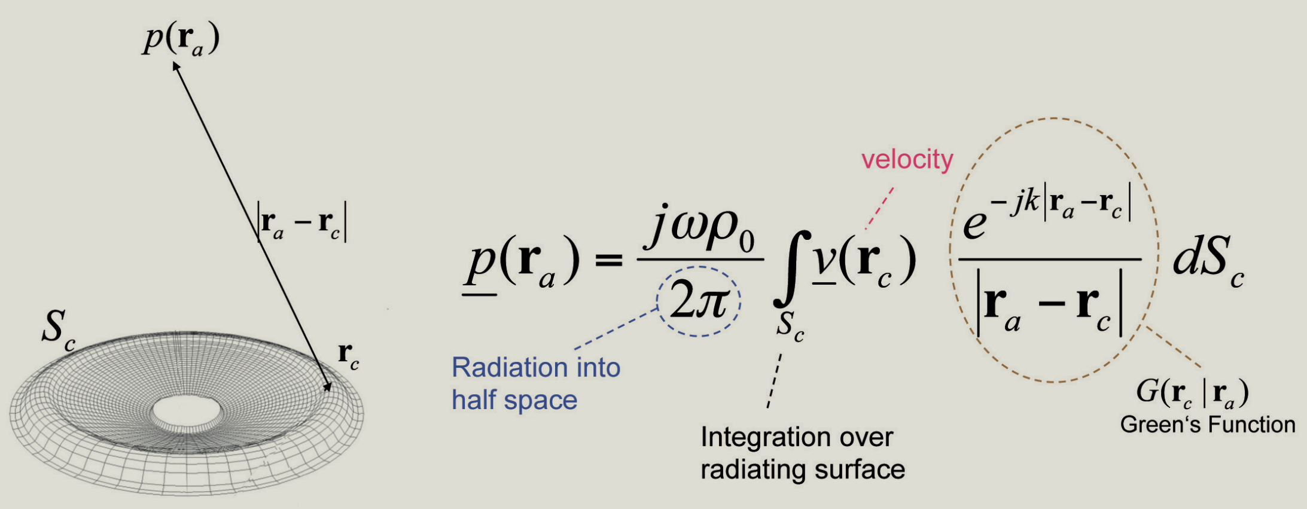


$$G(\mathbf{r}_a\,|\,\mathbf{r}_c)=\frac{e^{-jk|\mathbf{r}_a-\mathbf{r}_c|}}{|\mathbf{r}_a-\mathbf{r}_c|}$$

$$\underline{p}(\mathbf{r}_a)=\frac{1}{4\pi}\iint_{S_c}\Big(\underbrace{j\omega\rho_0 G(\mathbf{r}_c\,|\,\mathbf{r}_a)\,\underline{v}_n(\mathbf{r}_c)}_{\text{Monopoles}}-\underbrace{\underline{p}(\mathbf{r}_c)\frac{\partial}{\partial n}G(\mathbf{r}_c\,|\,\mathbf{r}_a)}_{\text{Dipoles}}\Big)dS_c$$

Sound pressure at observing point $\mathbf{r}_a$ can be predicted by monopole and dipole sources on the loudspeaker surface which satisfy the boundary condition of the moving cone and rigid enclosure.

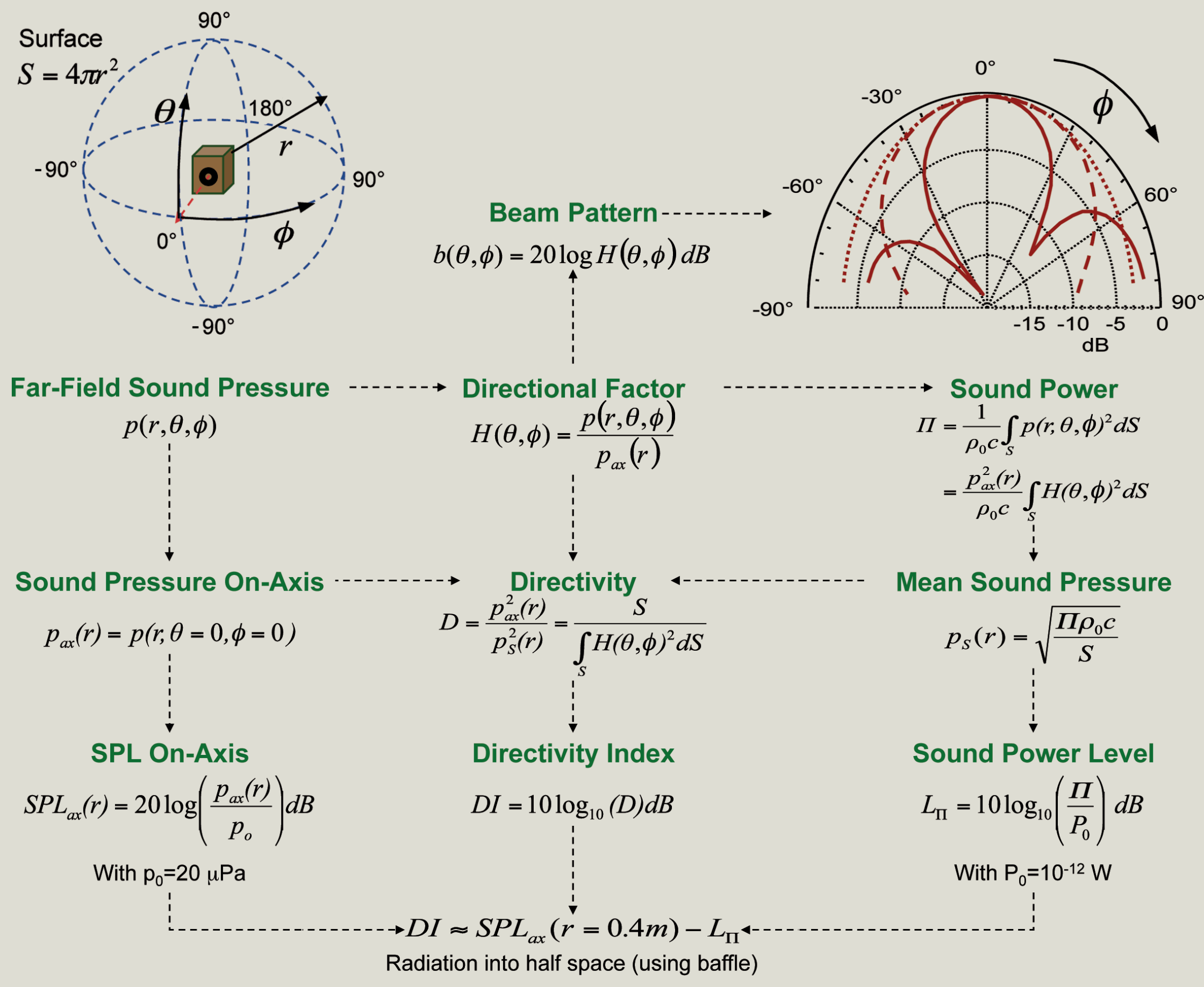
Monopole Approximation



$$\underline{p}(\mathbf{r}_a)=\frac{j\omega\rho_0}{2\pi}\int_{S_c}\underline{v}(\mathbf{r}_c)\frac{e^{-jk|\mathbf{r}_a-\mathbf{r}_c|}}{|\mathbf{r}_a-\mathbf{r}_c|}dS_c$$

Rayleigh's integral is a fast alternative to the BEM for predicting the sound pressure in the far field using monopoles only. The Green's function describes the phase shift and attenuation of the contribution due to the distance between source point $\mathbf{r}_c$ and the observing point $\mathbf{r}_a$. The monopole approximation is sufficiently accurate for shallow radiators operated in an infinite baffle.

Directional Characteristics



Beam Pattern

$$b(\theta,\phi)=20\log H(\theta,\phi)\,dB$$

Far-Field Sound Pressure

$$p(r,\theta,\phi)$$

Directional Factor

$$H(\theta,\phi)=\frac{p(r,\theta,\phi)}{p_{ax}(r)}$$

Sound Power

$$\Pi=\frac{1}{\rho_0 c}\int_S p(r,\theta,\phi)^2 dS=\frac{p_{ax}^2(r)}{\rho_0 c}\int_S H(\theta,\phi)^2 dS$$

Sound Pressure On-Axis

$$p_{ax}(r)=p(r,\theta=0,\phi=0)$$

Directivity

$$D=\frac{p_{ax}^2(r)}{p_S^2(r)}=\frac{S}{\int_S H(\theta,\phi)^2 dS}$$

Mean Sound Pressure

$$p_S(r)=\sqrt{\frac{\Pi\rho_0 c}{S}}$$

SPL On-Axis

$$SPL_{ax}(r)=20\log\left(\frac{p_{ax}(r)}{p_o}\right)dB$$

With p_0=20 μPa

Directivity Index

$$DI=10\log_{10}(D)dB$$

Sound Power Level

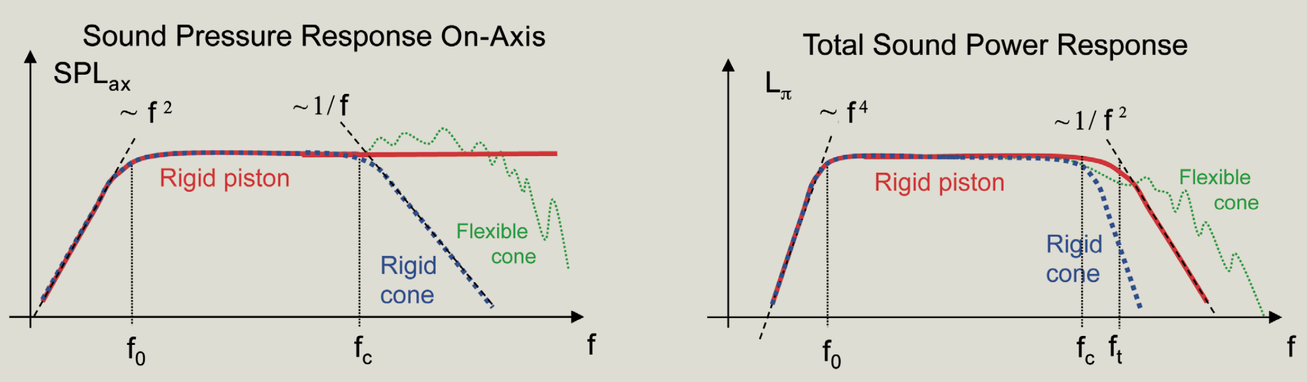
$$L_\Pi=10\log_{10}\left(\frac{\Pi}{P_0}\right)dB$$

With P_0=10^{-12} W

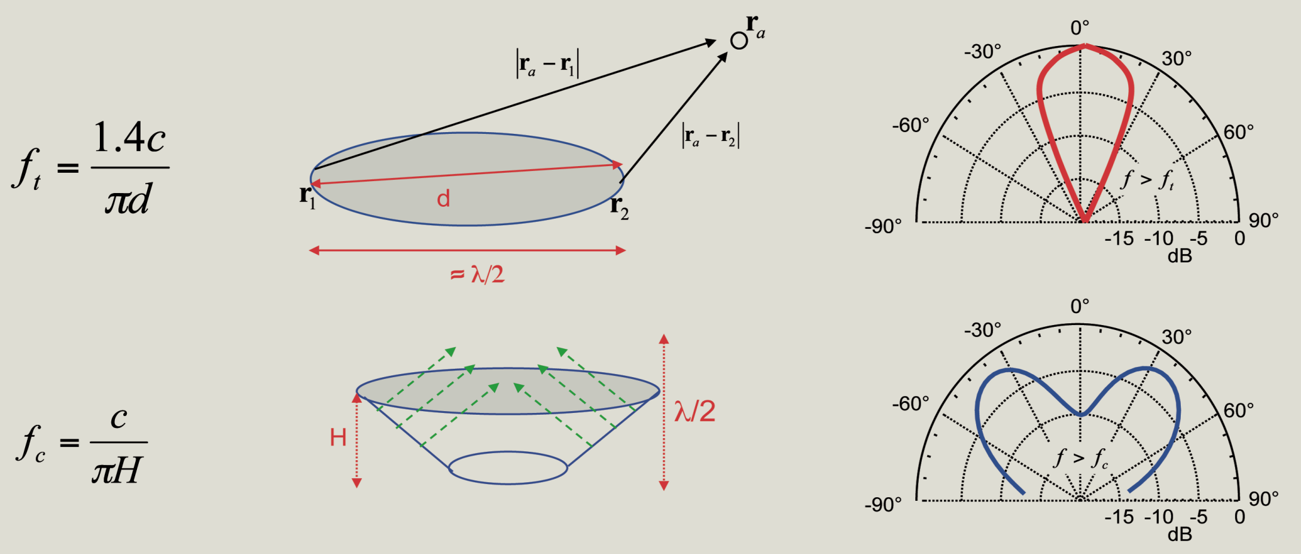
$$DI\approx SPL_{ax}(r=0.4m)-L_\Pi$$

Radiation into half space (using baffle)

Bandwidth of Rigid Radiators



As long as the geometrical dimensions of the radiatior are small compared to the wavelength, the sound pressure in the far field is proportional to the acceleration of the radiator's surface. At low frequencies, where the driving force moves the mechanical suspension the sound pressure decreases by 12 dB per octave. Above the fundamental resonance frequency f_0 the driving force generates a constant acceleration of the moving mass and a constant sound pressure output. Above the transfer frequency f_t where the diameter *d* of the radiator exceeds half of the wavelength, the total sound power decreases by 6 dB per octave. The sound pressure on-axis of the rigid cone decreases by 6 dB per octave above the cone cut-off frequency f_c where the cone elements have a larger distance than half the wavelength due to the cone height *H*.

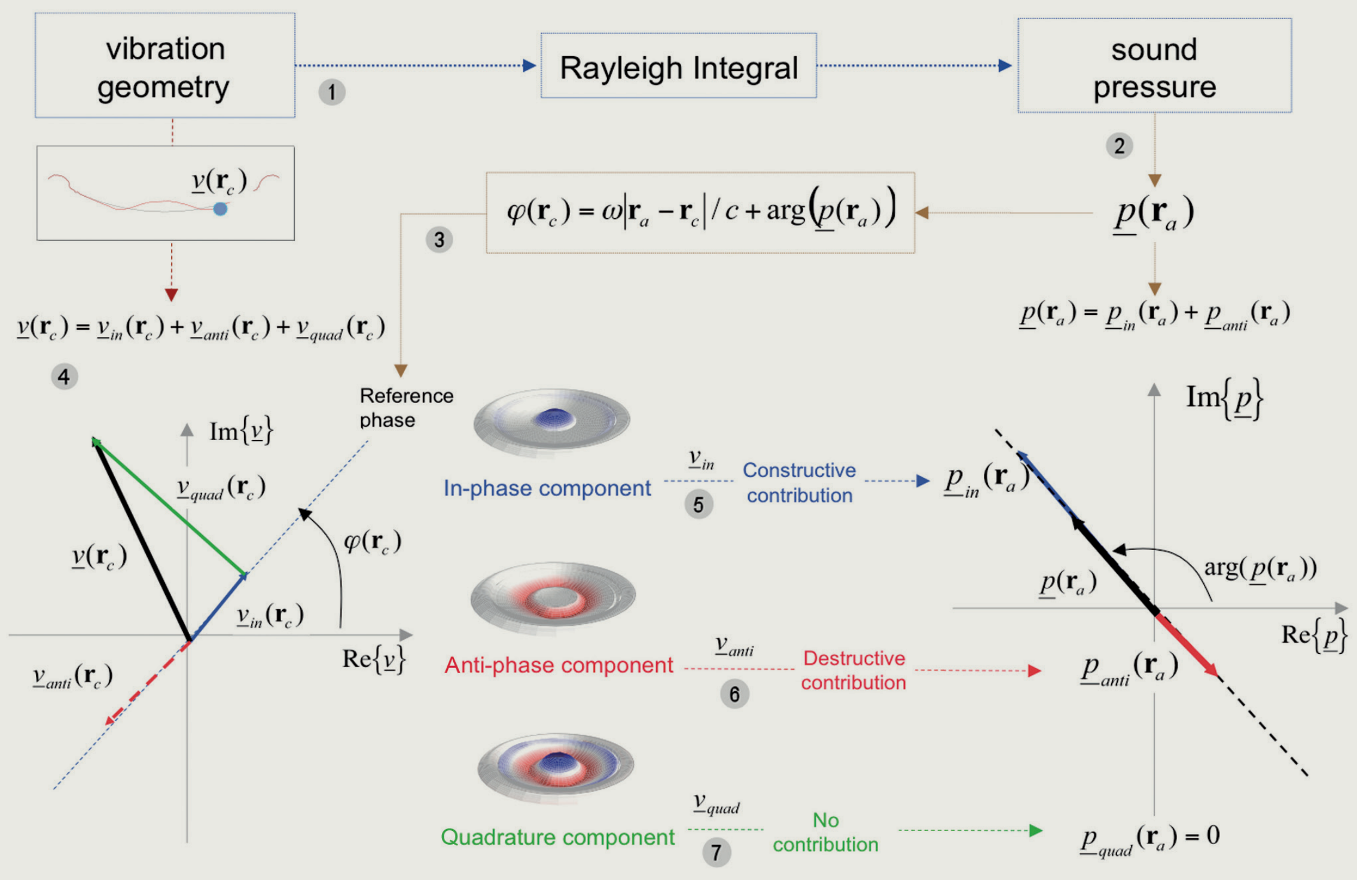


$$f_t=\frac{1.4c}{\pi d}$$

$$f_c=\frac{c}{\pi H}$$

ANALYSIS

Sound-Pressure-Related Decomposition



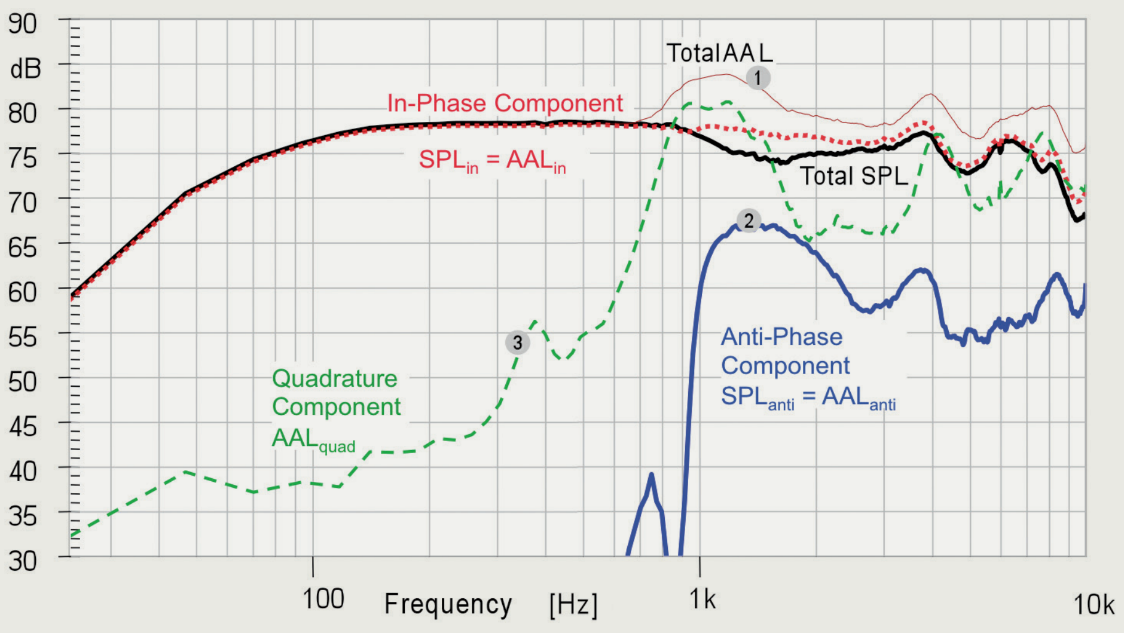
$$\varphi(\mathbf{r}_c)=\omega|\mathbf{r}_a-\mathbf{r}_c|/c+\arg(\underline{p}(\mathbf{r}_a))$$

$$\underline{v}(\mathbf{r}_c)=\underline{v}_{in}(\mathbf{r}_c)+\underline{v}_{anti}(\mathbf{r}_c)+\underline{v}_{quad}(\mathbf{r}_c)$$

$$\underline{p}(\mathbf{r}_a)=\underline{p}_{in}(\mathbf{r}_a)+\underline{p}_{anti}(\mathbf{r}_a)$$

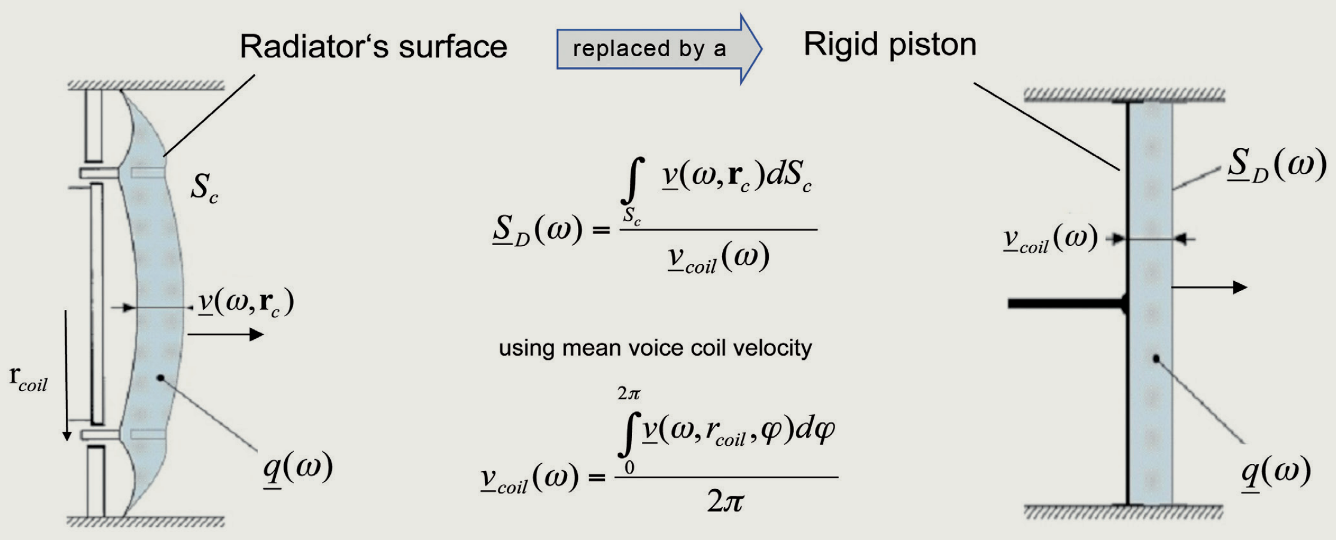
1. Laser scanning provides the amplitude and phase of the mechanical vibration (e.g. velocity) and the precise position of the source point $\mathbf{r}_c$ at the surface of the radiator.
2. The Rayleigh integral is used to calculate the amplitude and phase of the sound pressure at the observing point $\mathbf{r}_a$.
3. The phase arg(p($\mathbf{r}_a$)) of the resulting sound pressure is transformed as a reference phase to each source point $\mathbf{r}_c$ at the surface of the radiatior considering the distance between the two points.
4. The total mechanical vibration is split into an in-phase component which is in phase with the reference phase or an anti-phase component in the opposite direction and a quadrature component which is 90 degree shifted to the reference phase.
5. The total in-phase component $v_{in}(\mathbf{r}_c)$ accumulated over all points $\mathbf{r}_c$ on the radiator's surface is the constructive contribution $p_{in}(\mathbf{r}_a)$ to the sound pressure at the observing point $\mathbf{r}_a$.
6. The total anti-phase components $v_{anti}(\mathbf{r}_c)$ represents the destructive contribution $p_{anti}(\mathbf{r}_a)$ to the total sound pressure.
7. The total quadrature component $v_{quad}(\mathbf{r}_c)$ describes the vibration which has no effect on the sound pressure at observing point $\mathbf{r}_a$.

Sound Pressure Components



1. The in-phase component may be larger than the total SPL but never exceeds the total AAL. These curves coincide below cone break-up where the anti-phase and quadrature components are negligible.
2. The anti-phase component generates identical values of SPL_{anti} and the AAL_{anti} which rise rapidly at the break-up frequency but never exceed the values SPL_{in} and AAL_{in} of the in-phase component. However, a small difference between in-phase and anti-phase component causes a dip in the total SPL (see acoustical cancellation).
3. The quadrature component produces no sound pressure but the AAL_{quad} may exceed the in-phase component. However, it is never larger than the total AAL. The peak at 380 Hz indicates a rocking mode.

Effective Radiation Area S_D

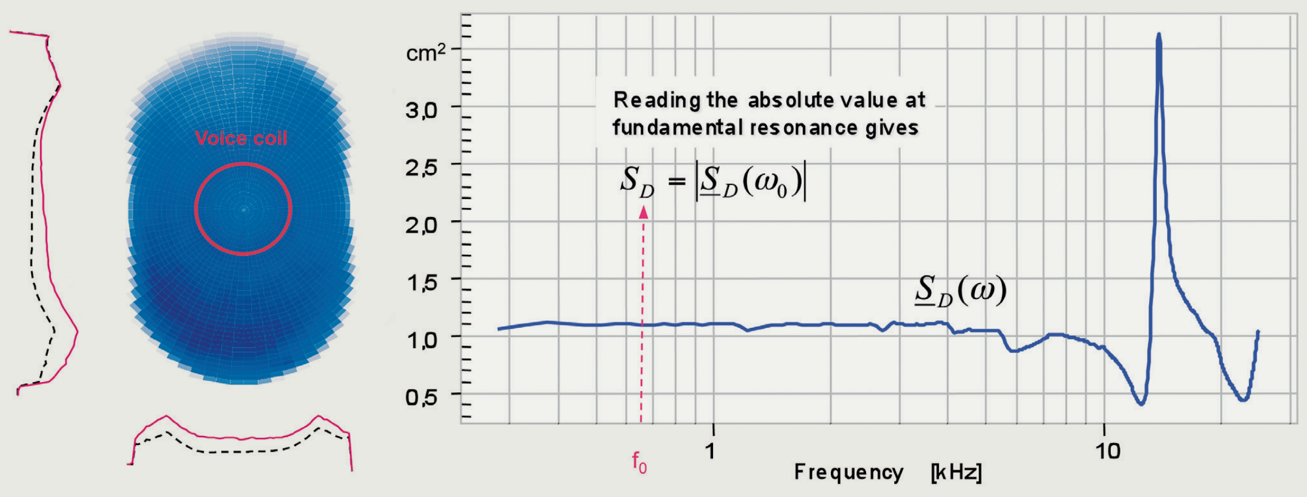


$$\underline{S}_D(\omega)=\frac{\int_{S_c}\underline{v}(\omega,\mathbf{r}_c)dS_c}{\underline{v}_{coil}(\omega)}$$

using mean voice coil velocity

$$\underline{v}_{coil}(\omega)=\frac{\int_0^{2\pi}\underline{v}(\omega,r_{coil},\varphi)d\varphi}{2\pi}$$

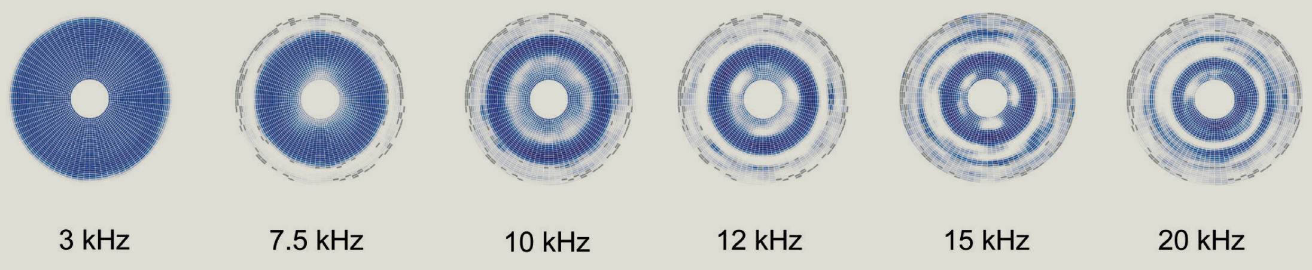
The effective radiation area S_D is an important lumped parameter describing the surface of a rigid piston moving with the mean value of the voice coil velocity v_{coil} and generating the same volume velocity *q* as the radiator's surface. The integration of the scanned velocity can cope with rocking modes and other asymmetrical vibration profiles.



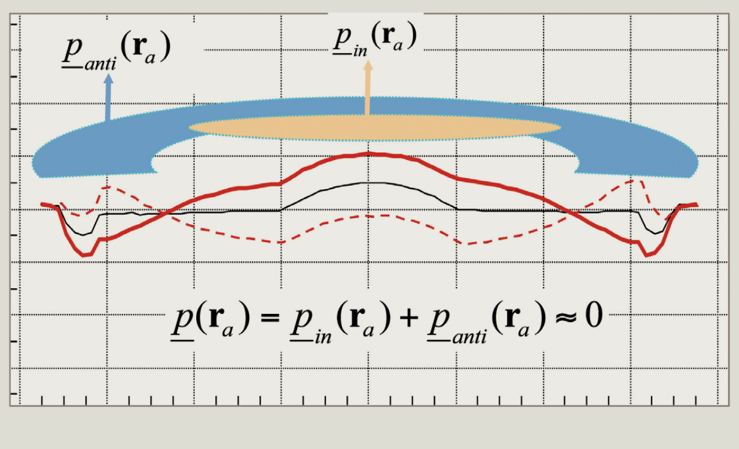
DIAGNOSTICS

Shrinking Radiation Area?

Cone radiators with a lower bending stiffness at the outside area will generate the first anti-phase and quadrature components there. The in-phase component of these radiators is always located in the center of the cone, which shrinks in size at higher frequencies. Thus a woofer with a flexible cone has a smaller directivity index at higher frequencies than a rigid cone.

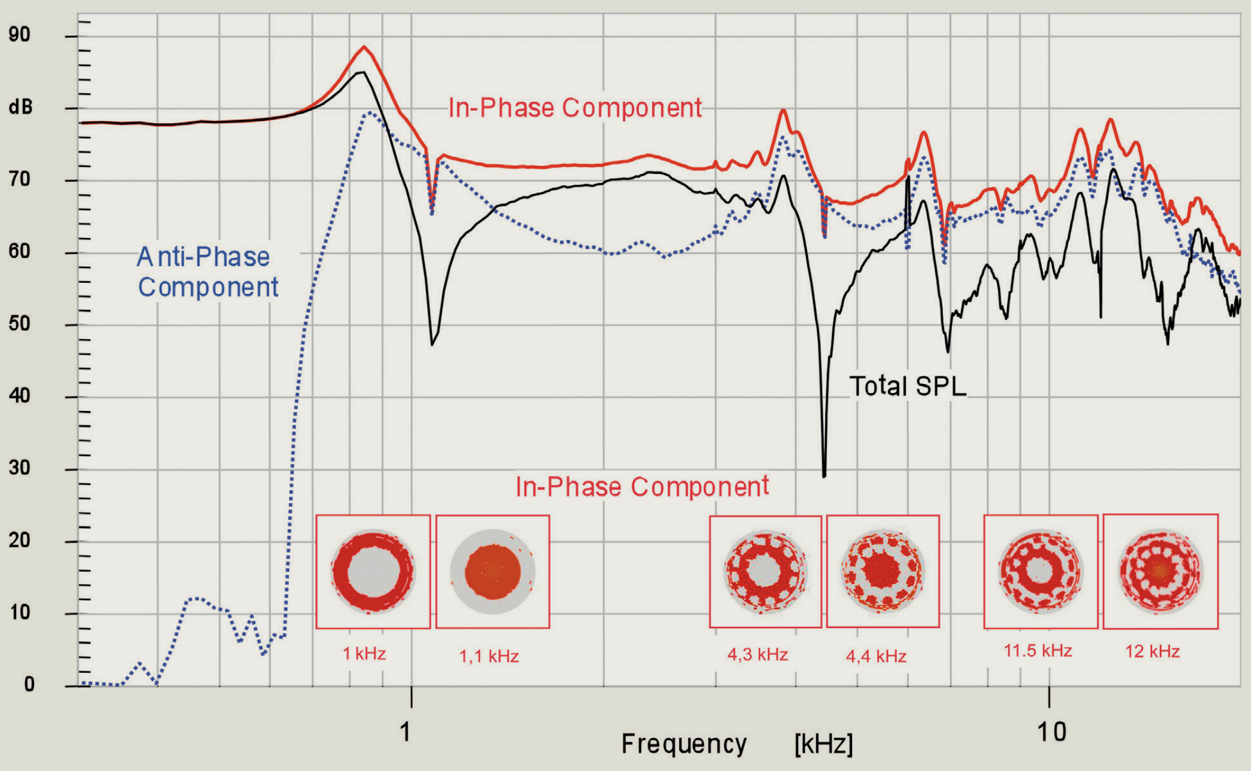


Acoustical Cancellation?

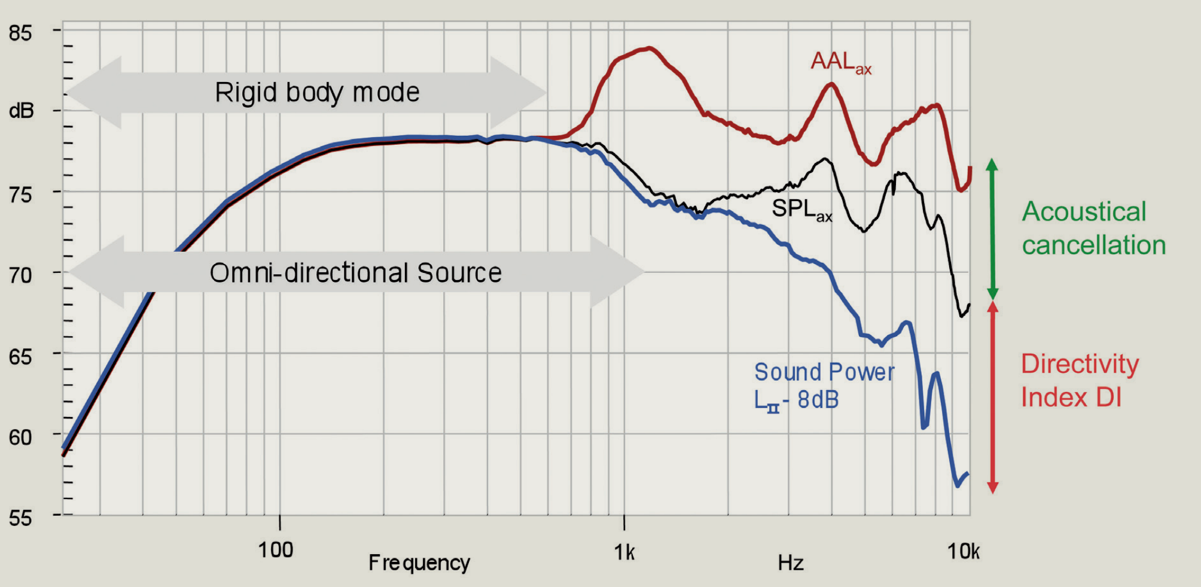


$$\underline{p}(\mathbf{r}_a)=\underline{p}_{in}(\mathbf{r}_a)+\underline{p}_{anti}(\mathbf{r}_a)\approx 0$$

Significant dips in the SPL response are caused by acoustical cancellation which occurs if the anti-phase component is not negligible and the difference between in-phase component SPL_{in} and anti-phase component SPL_{anti} becomes smaller than 10 dB. At the cancellation frequency the in-phase and anti-phase components exchange their location on the radiator's surface as shown below.

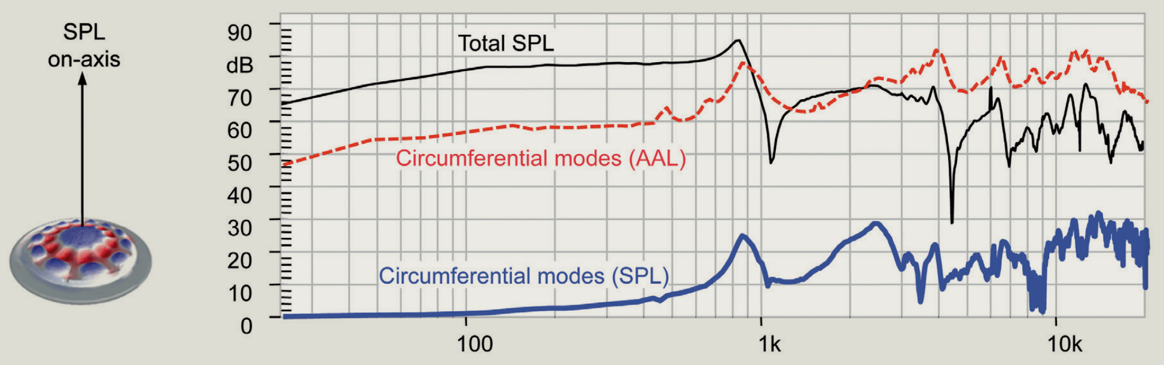


Desired Directivity?

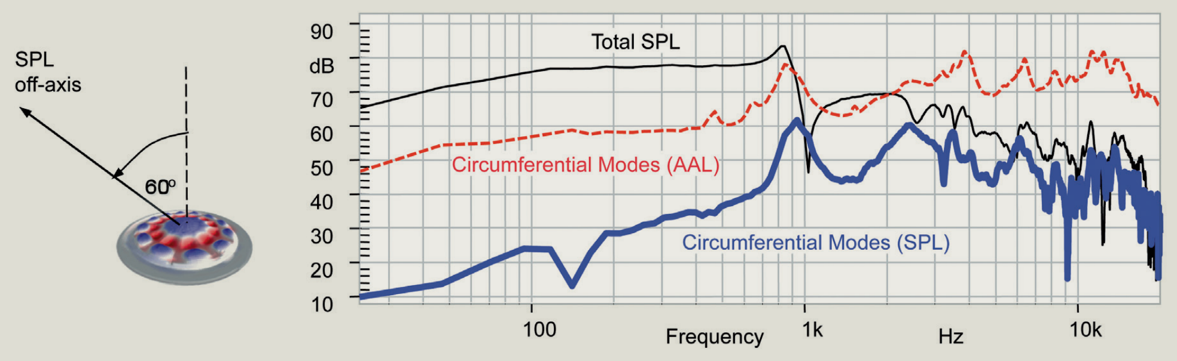


Three curves reveal the most important radiation properties: Accumulated acceleration AAL_{ax}, sound pressure SPL_{ax} on axis and sound power response L_Π have the identical curve shape below cone break-up where the geometrical dimensions are much smaller than the aocustical wavelength. The difference between AAL_{ax} and SPL_{ax} describes the acoustical cancellation and the difference between SPL_{ax} and sound power response L_Π corresponds with the directivity index.

Contribution of Circumferential Modes?



Circumferential modes having high AAL produce low SPL on-axis but contribute significantly to the SPL off-axis and to the total sound power at higher frequencies.



Glossary of Symbols:

$\omega=2\pi f$	Angular frequency	$\lvert\underline{p}\rvert$	Amplitude (peak value) of complex sound pressure amplitude
c	Speed of sound	$\arg(\underline{p})$	Argument (phase) of complex sound pressure
ρ_0	Density of air	$\mathrm{Im}\{\underline{p}\}$	Imaginary part of complex sound pressure amplitude
$k=\omega/c$	Wave number	$\mathrm{Re}\{\underline{p}\}$	Real part of complex sound pressure amplitude
$\lambda=c/f$	Wave length	$p=\lvert\underline{p}\rvert/\sqrt{2}$	rms value of sound pressure

References:

1. W. Klippel, J. Schlechter, „Distributed Mechanical Parameters of Loudspeakers Part 1: Measurements", Journal of Audio Eng. Soc. Vol. 57, No. 7/8, 2009 July/August, page 500 – 511
2. W. Klippel, J. Schlechter, „Distributed Mechanical Parameters of Loudspeakers Part 2: Diagnostics", Journal of Audio Eng. Soc. Vol. 57, No. 9, 2009, page 696 – 708
3. F. J. M. Frankort, „Vibration Patterns and Radiation Behavior of Loudspeaker Cones", J. Audio Eng. Soc., vol. 26, pp. 609–622 (1978 September)
4. A. Kaizer and A. Leeuwestein, „Calculation of the Sound Radiation of a Nonrigid Loudspeaker Diaphragm Using the Finite-Element Method", J. Audio Eng. Soc., vol. 36, pp. 539–551 (1988 July/August)
5. W. Klippel, J. Schlechter, „Dynamical Measurement of the Effective Radiation Area SD", Journal of Audio Eng. Soc.
6. L. Kinsler, et. Al, „Fundamentals of Acoustics, John Wiley & Sons, Inc. 2000
7. L. Beranek, Acoustics (Acoustic Society of America, New York, 1996)

www.klippel.de